Neutrino Oscillations

and much, much, much, much more

OVERVIEW

- ✧ Neutrino Oscillations
- ✧ MSW (Matter Effect)
- ✧ RSFP (Magnetic Effect)
- ✧ MSW + RSFP = ?
- ✧ ME

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Neutrino Eigenstates

Flavor States

$\nu_e =$

$\nu_\mu =$

$\nu_\tau =$

Neutrino Eigenstates

Mass States

$\nu_1 =$

$\nu_2 =$

$\nu_3 =$

Neutrino Eigenstates

Mass States

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$\nu_2 =$

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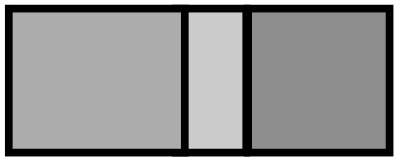
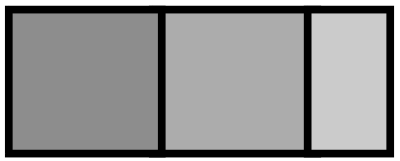
Flavor States

$\nu_e =$

$\nu_\mu =$ 

$\nu_\tau =$ 

Neutrino Eigenstates

Mass States

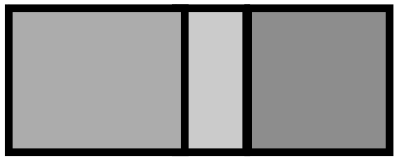
$\nu_1 =$

$\nu_2 =$

$\nu_3 =$

Flavor States

$\nu_e =$

$\nu_\mu =$ 

$\nu_\tau =$

How to describe neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

How to describe neutrino mixing

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$$\nu_e = \cos\theta \nu_1 + \sin\theta \nu_2$$

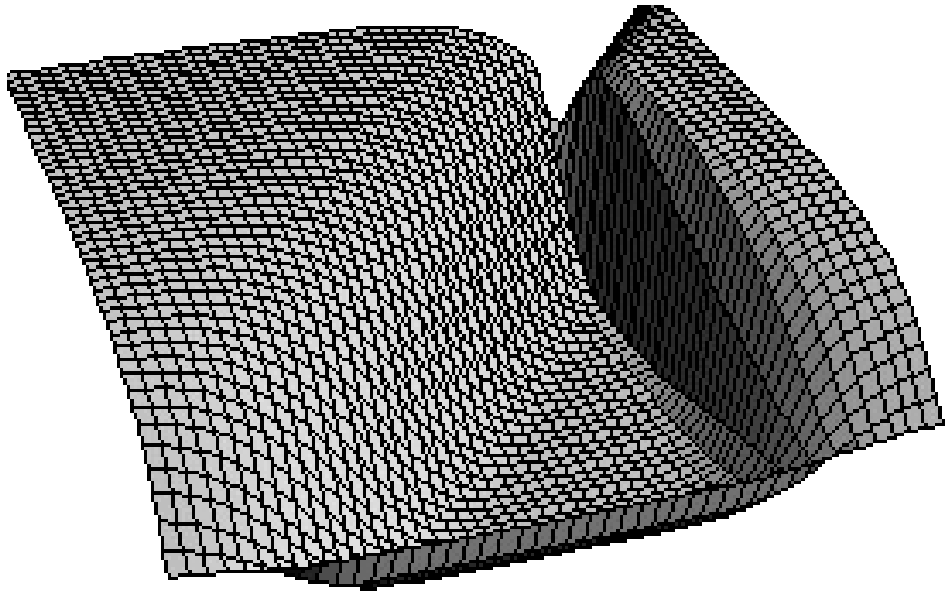
$$\nu_\mu = -\sin\theta \nu_1 + \cos\theta \nu_2$$

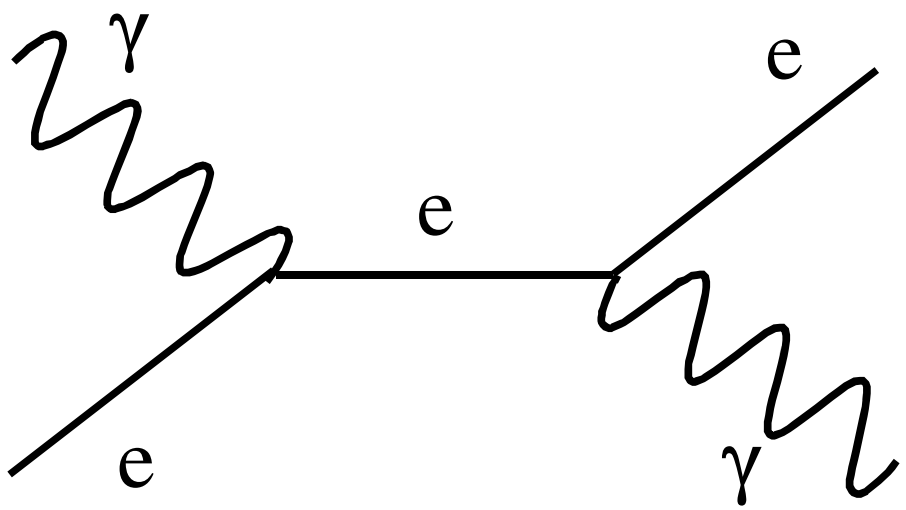
Vacuum Oscillations

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\pi x}{\lambda}\right)$$

$$\lambda = \frac{2.5E}{\Delta m^2}$$

The Mikheyev-Smirnov-Wolfenstein Effect (MSW)

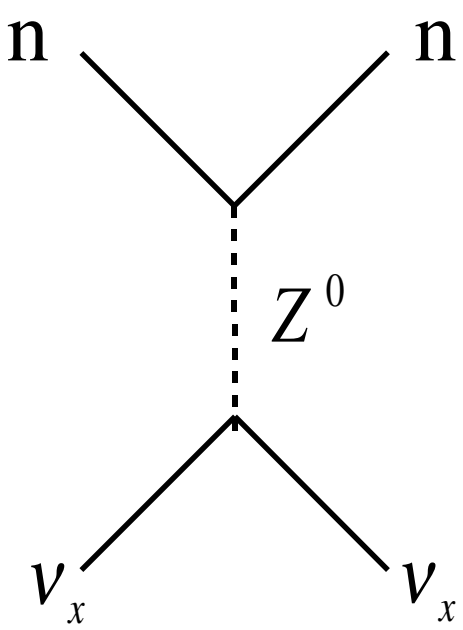




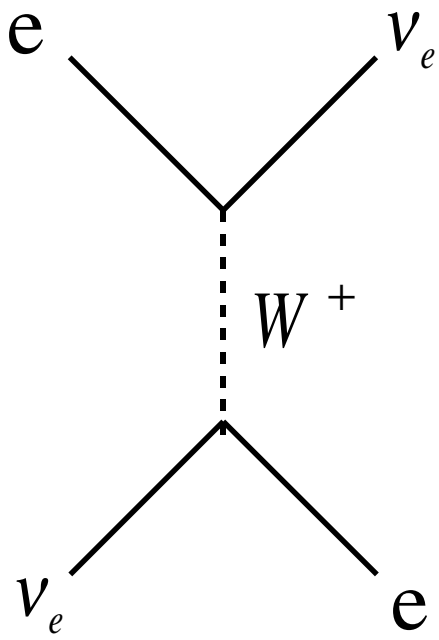
The speed of light in a medium:

$$c_m = \frac{c}{n}$$

Neutral Current



Charged Current



MSW Transition Probability:

$$P(\nu_{el} \to \nu_{\mu l}) = \frac{1}{2} - \frac{1}{2}\cos 2\theta \cos 2\theta_m (1 - 2P_{hop})$$

$$P_{hop} = \exp\left(-\frac{\pi}{2}\frac{\sin^2 2\theta}{\cos 2\theta}\frac{\Delta m^2}{2E}\frac{1}{\left|\frac{1}{\rho}\frac{d\rho}{dr}\right|_{r=r_c}}\right)$$

Resonant Spin Flavor Precession: (RSFP)

Dirac Neutrinos

Majorana Neutrinos

Schrodinger's Equation

$$i\frac{d}{dt}\mathbf{A} = \mathbf{H}\mathbf{A}$$

$$\mathbf{A} = \begin{pmatrix} \nu_{el} \\ \nu_{\mu l} \\ \nu_{er} \\ \nu_{\mu r} \end{pmatrix}, \quad \mathbf{H} = Hamiltonian$$

Dirac

$$\begin{pmatrix} \frac{\Delta m^2}{2E}\sin^2\theta + a_e & \frac{\Delta m^2}{4E}\sin 2\theta & \mu_{ee}B & \mu_{e\mu}B \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos^2\theta + a_\mu & \mu_{\mu e}B & \mu_{\mu\mu}B \\ \mu_{ee}B & \mu_{e\mu}B & \frac{\Delta m^2}{2E}\sin^2\theta & 0 \\ \mu_{\mu e}B & \mu_{\mu\mu}B & 0 & \frac{\Delta m^2}{2E}\cos^2\theta \end{pmatrix}$$

Majorana

$$\begin{pmatrix} \frac{\Delta m^2}{2E}\sin^2\theta + a_e & \frac{\Delta m^2}{4E}\sin 2\theta & 0 & \mu B \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos^2\theta + a_\mu & \mu B & 0 \\ 0 & \mu B & \frac{\Delta m^2}{2E}\sin^2\theta - a_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \mu B & 0 & \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos^2\theta - a_\mu \end{pmatrix}$$

Basis: $\left(\nu_{el}, \nu_{\mu l}, \nu_{er}, \nu_{\mu r}\right)$

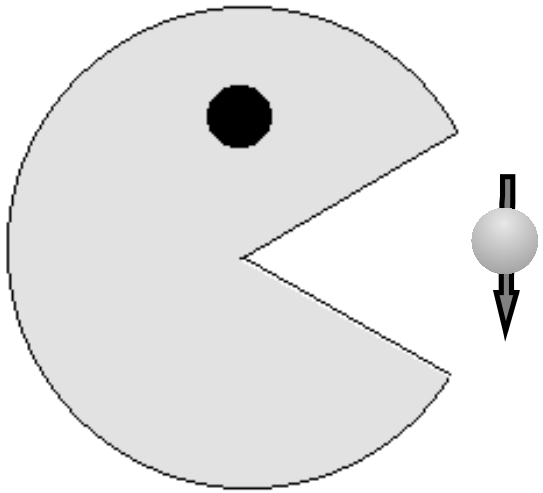
$$a_e = \frac{G_F}{\sqrt{2}}\frac{11}{6}N_e \ , \quad a_\mu = -\frac{G_F}{\sqrt{2}}\frac{1}{6}N_e$$

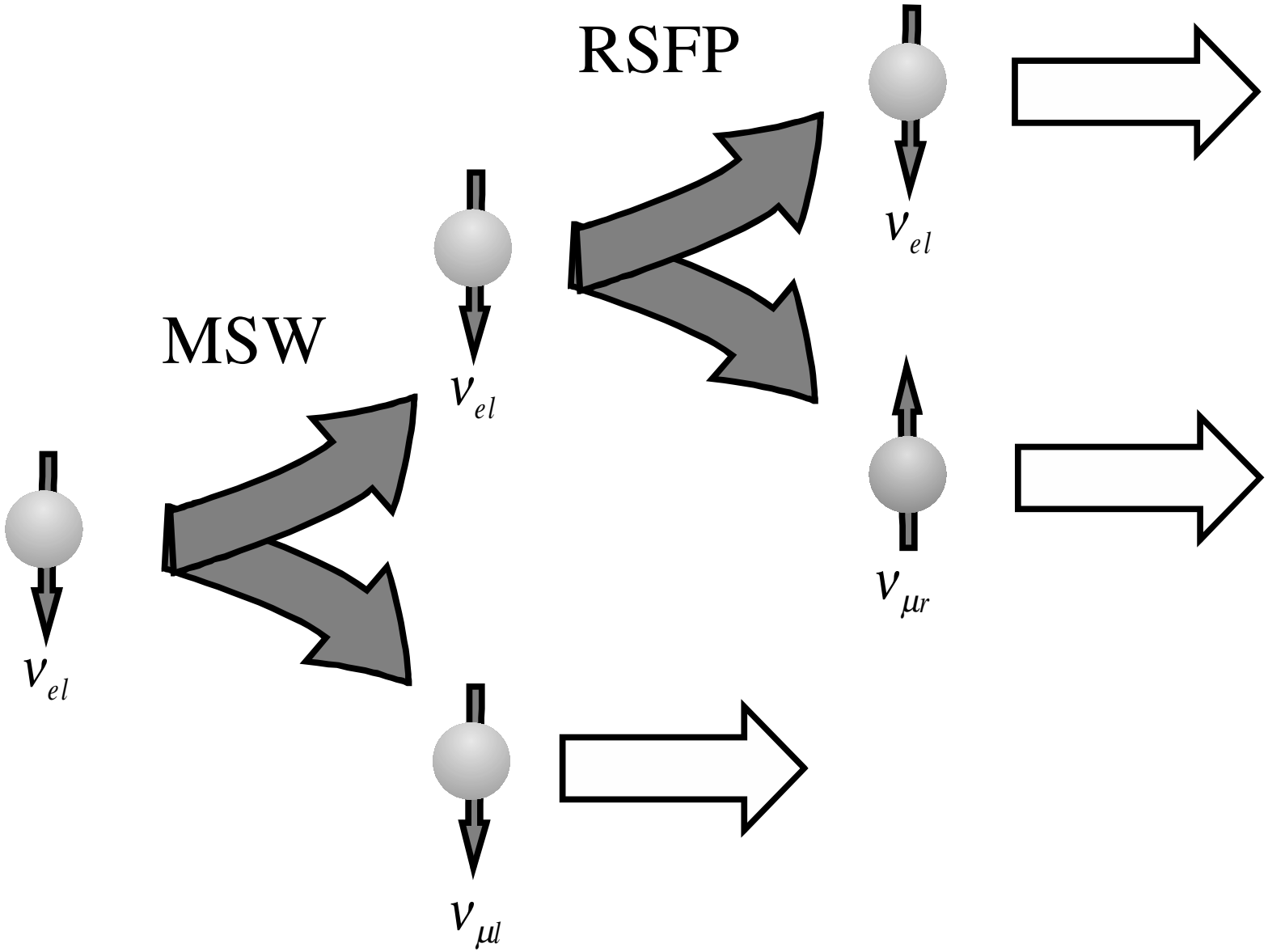
RSFP Transition Probability:

$$P(\nu_{el} \rightarrow \nu_{\mu r}) = \frac{1}{2} - \frac{1}{2}\cos 2\theta_i \cos 2\theta_f (1 - 2P_{LZ})$$

$$P_{LZ} = \exp\left(-\frac{2\pi(\mu B)^2}{\cos^2\theta \dfrac{\Delta m^2}{2E}} \frac{1}{\left|\frac{1}{\rho}\frac{d\rho}{dr}\right|_{r=r_c}}\right)$$

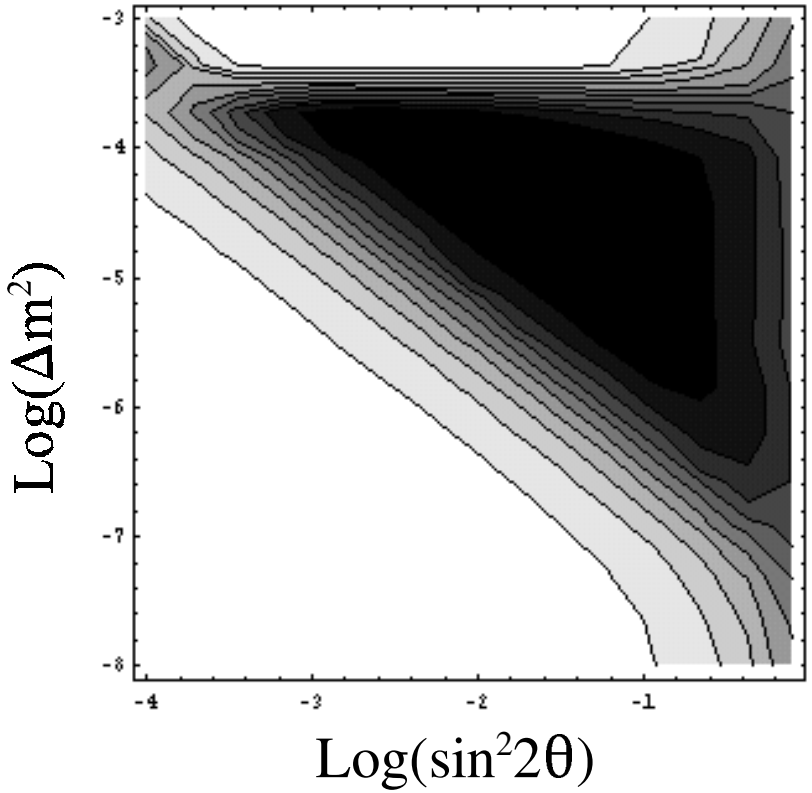
The Combined Effects:




RSFP
MSW
ν_{el}
ν_{el}
ν_{el}
$\nu_{\mu r}$
$\nu_{\mu l}$

Mathematica Modules

MSW



RSFP

dm^2	Pad	Plz	P
2.42E-05	0.000	0.802	0.000
1.17E-05	0.214	0.633	0.214
1.06E-05	0.998	0.604	0.396
5.90E-06	1.000	0.404	0.596
3.36E-06	1.000	0.204	0.796
1.17E-06	1.000	0.010	0.990
7.90E-10	0.991	0.000	0.991
4.07E-10	0.873	0.000	0.873
2.40E-10	0.218	0.000	0.218
2.10E-10	0.146	0.000	0.146
0	0.022	0.000	0.022

Transition Probabilities

The next step...

Russian-American Gallium Experiment
(SAGE)

$$^{71}Ga + \nu_e \rightarrow {}^{71}Ge + e^-$$

+

Solar Neutrino Spectrum

=

Bounds for μB, Δm^2, and θ

Thanx to:

Dr. Amiri
Dr. Balantekin
Dr. Carroll
Dr. Elliot
Dr. Haxton
Dr. Pulido
Dr. Schroeder

The NSF

The letters ל and Ж and
the non-number ∞

And... *F. A. Kimball*