Effective Field Theory and Deuteron EM Properties

Matthew P. Dorsten
Advisor: D.R. Phillips
University of Washington/INT
REU99

27 August 1999

Abstract

I give an elementary introduction to effective field theory, followed by background for the calculation of EM form factors of the deuteron. Some simple model calculations illustrate the connection between the two, and then I discuss realistic results and an extension to higher order of the calculation in Ref. [2].

1 Introduction to Effective Field Theory

Quantum Chromodynamics (QCD) is the field theory that describes nuclear physics. It can, in principle, be solved for all the information relevant to the interactions between nucleons. However, this could only be done numerically, and the most advanced calculations to date only deal with single nucleons. The prospects for direct computer simulations of NN scattering are slim. Describing real nuclei with this approach would be phenomenally difficult. In addition, calculations in terms of quarks and gluons are vastly inefficient at low energies, where the natural degrees of freedom are nucleons.

An alternative to solving the full theory is using an effective one. Much of nuclear physics traditionally has been formulated in terms of potential models involving several parameters fit to data, and this largely phenomenological approach has been very fruitful. Results typically agree with experiment to a few percent. However, this approach is limited by the amount of QCD physics that it includes in the effective potential. Since external currents, among other things, are difficult to include, some observables are not as accurate as others. Also, inelastic processes are hard to model.

Effective field theories (EFT) are a different sort of low-energy approximation. Renormalization theory says that low-energy processes are insensitive to the details of high-energy dynamics [4]. This means that a theory containing the true long distance physics is a good representation of the true theory at low energies, regardless of its short distance details. Recalling the inverse relationship between momentum and wavelength, this is analogous to the fact that a probe with wavelength λ is unable to discern structure with a characteristic length much smaller than λ. If, for example, the target is some charge distribution, it can be approximated by a number of multipole terms depending on λ.

For this to make sense, we need a clear hierarchy of scales in the problem. For nuclear physics, we have $m_\pi \ll m_\rho$. Thus, pion physics, such as one-pion exchange, may be considered low-energy, while physics occurring at the scale of m_ρ is treated as high-energy.

Effective field theories use chiral perturbation theory to substitute a relatively simple effective Lagrangian for the true one, which may be very complicated. This takes the form of a series of nucleon-nucleon contact interactions, the coefficients of which are determined from experimental data or, as may be possible in the near future, lattice QCD calculations. Eq. (1) is an example of the S-wave piece of the series of nucleon-nucleon contact interactions [2].

$$\mathcal{L}_{\mathrm{NN,eff}} = C_0(N^\dagger N) + C_2[(N^\dagger N)(N^\dagger \mathbf{D}^2 N) + (N^\dagger N)(\mathbf{D}^2 N^\dagger N)] + \cdots \tag{1}$$

It is C_0 and C_2 that need to be determined.

Some regularization scheme is required to render finite the loop integrals prescribed by this effective Lagrangian. The most intuitive method is cutoff regularization, which essentially cuts off the integrals at some momentum scale Λ. This momentum scale indicates what is being treated as high energy, and it ensures that the effective Lagrangian is not used at high energy, where it cannot be valid. This limit will manifest itself as the breakdown scale for calculated observables, that is, where the results differ significantly from experiment. Dimensional regularization is another popular scheme. Loop integrals are done in D dimensions, say, and the infinities that arise in four dimensions are subtracted off. Then the limit of the result is taken as D approaches four.

2 Review of Form Factors

Electron scattering is a powerful tool for probing charge distributions. Varying the momentum transferred to the target for a fixed energy loss, a capability lacking in photon scattering, yields a profile of the target [1]. This profile is embodied in quantities known as "form factors." For a point-like target, the form factor is unity, so the form factor measures the structure of the target. The actual relation is

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} |F(q)|^2 \quad , \tag{2}$$

where the Mott cross section refers to a point-like target [3]. For a static, spinless target, the form factor is just the Fourier transform of the charge distribution:

$$F(\mathbf{q}) = \int d^3x\, e^{i\mathbf{q}\cdot\mathbf{x}}\,\rho(\mathbf{x}) \ . \tag{3}$$

However, because the deuteron lacks spherical symmetry (i.e., it has D-states), there are other terms in the form factor. These correspond to higher terms in a multipole expansion of the charge distribution. In addition, the distribution is not static, so there is a multipole expansion of the current and, consequently, magnetic form factors. In a relativistic treatment, these form factors are gotten from matrix elements of the deuteron electromagnetic current to which a virtual photon couples [2].

3 Illustration with Toy Deuteron

The method of separating the long and short scales in the deuteron can be clearly illustrated with elementary quantum mechanics. A naïve model is a square well with the depth V_0 that gives the correct binding energy of the deuteron (γ^2/M) and a varying width R_0 :

$$V^{(0)}(r) = \begin{cases} -V_0 & \text{if } r \leq R_0 \\ 0 & \text{if } r > R_0 \end{cases} \quad . \tag{4}$$

By varying the width of the well by small amounts (i.e., changing the short-distance physics), we can test whether or not low-momentum observables are sensitive to short distance physics in an effective theory. The low-momentum observable of choice here is the slope at $q^2 = 0$ of the charge form factor, given in Eq. (3). This slope is proportional to the mean square radius, $<r^2>$, of the charge distribution [3]. This quantity is clearly a long-distance feature of the problem and should be relatively insensitive to the short-distance details of the potential.

However, this happens not to be the case. Because the tail of the wave function, $Ae^{-\gamma r}$, has a coefficient that varies as the width of the square well varies, the long-distance physics is still affected by short-distance physics. To remedy this situation, the entire tail of the wave function must be fixed, since the tail represents the long-distance physics. We clearly need another parameter in our potential to fix according to the experimental value of the coefficient A. This is achieved by adding to the square well a δ-shell, that is, a potential of the form

$$V^{(1)}(r) = -V_1 \delta(r - R_1) \ . \tag{5}$$

So the full potential is $V = V^{(0)} + V^{(1)}$, with V_0 and V_1 fit to γ and A. Fig. (1) shows that this procedure successfully separates short- and long-distance physics. The curves are very little different, especially at low momentum. This is shown more explicitly in

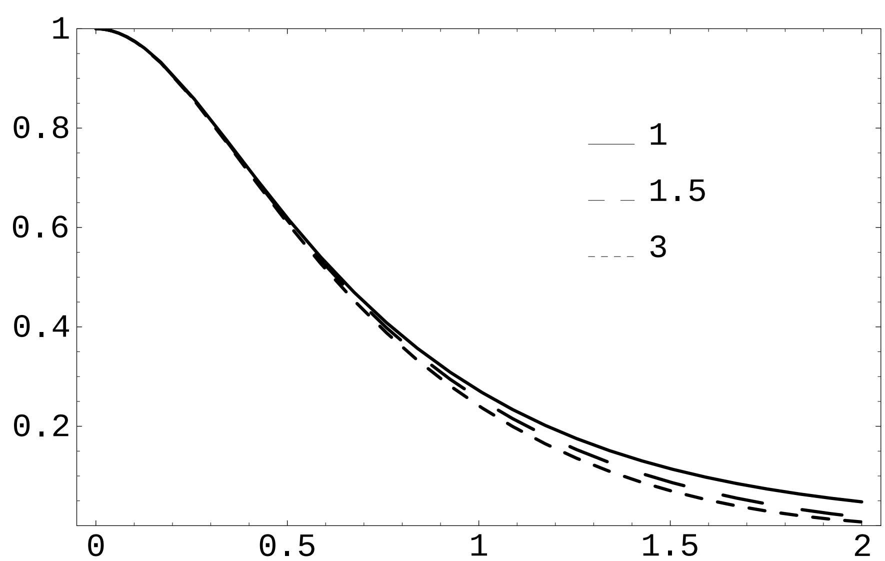


Figure 1: F_C vs. Q (fm^{-1}) for various values of the parameter R_1. For each curve, $R_0 = 3$ fm.

Fig. (2), where the error introduced by calculating the slope of F_C at $q^2 = 0$ with just the long-distance wave function is plotted against R_1 for fixed R_0. The error is small, which means that this truly is a long-distance quantity. In addition, it is relatively constant (on a reasonable scale), except for an irrelevant bump in the middle where the error changes sign. This supports the claim that the placement of the δ-shell does not significantly influence the long-distance physics.

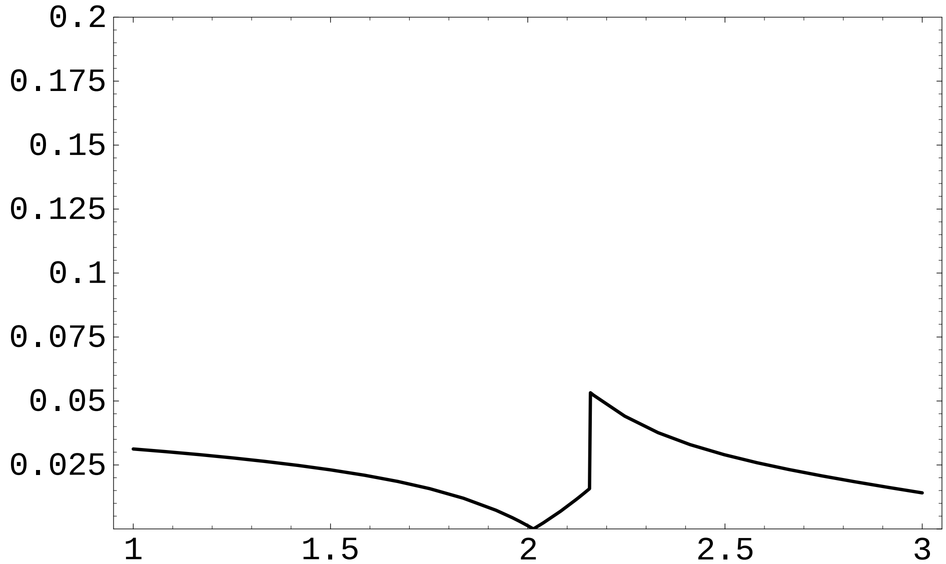


Figure 2: Relative error in slope at $q^2 = 0$ as a function of R_1 for R_0 fixed at 3 fm.

4 Realistic Deuteron

The process for getting realistic deuteron wave functions and, hence, form factors is very similar to that used in the toy calculation[1]. There are some details involved that complicate the matter slightly, such as including D-states, but the basic idea is the same. Once again, fixing the wave function tail is the most important part. This concentrates errors in the short-distance part, something required by EFT. There certainly are other ways of fitting the wave function, but those spread the errors throughout the wave function, mixing short- and long-distance physics.

For the pionless theory (i.e., the theory that treats pion physics as high energy), the idea is to integrate the free Schrödinger equation from $r = \infty$ to $r = R$, for some small radius R. R functions precisely as a momentum cutoff. In fact, using the language

[1]This section briefly summarizes the methods of Ref. [2].

introduced earlier, we can say $R \sim 1/\Lambda$. For smaller values of the radius, a short-distance regulator (e.g., a δ-shell) is used to prevent divergences at short distances. The resulting wave function is then used to calculate matrix elements of the current in order to find the various form factors. Fig. (3) shows the results for the quadrupole form factor using various values of the parameter R. They are compared with the

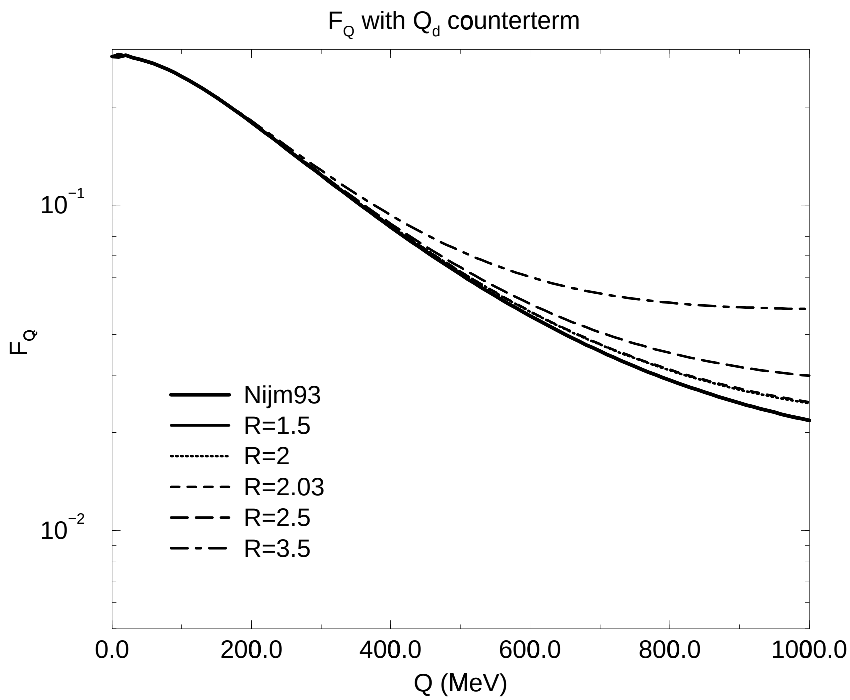
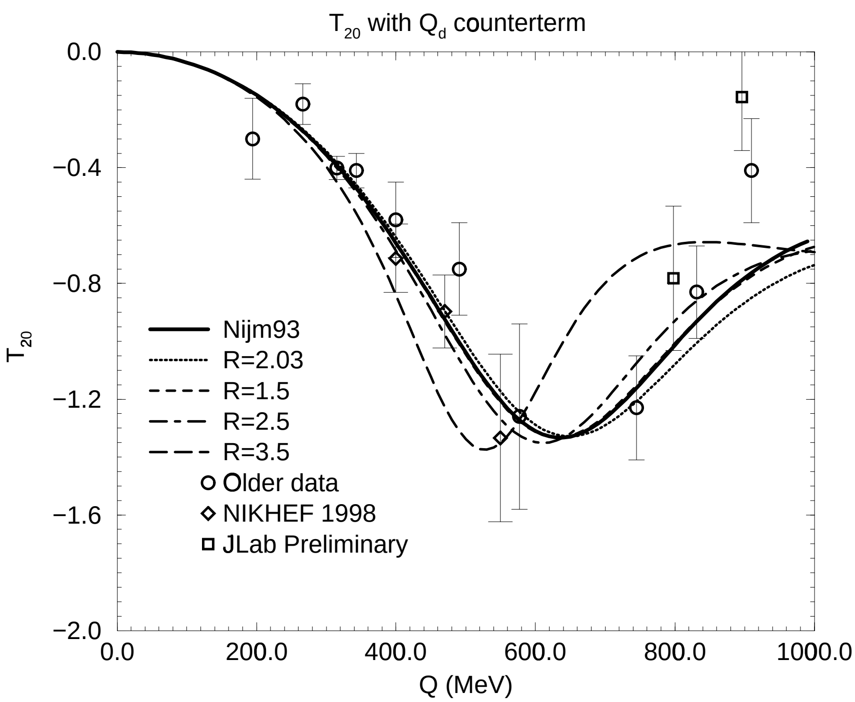


Figure 3: F_Q

Nijmegen potential-model calculations, which are very reliable. The results have been shifted at $Q = 0$ to match the experimental value, a process that amounts to adding a counterterm. This is necessary theoretically and substantially improves the results. The results are very good a low momentum, and they improve as R gets smaller. This is precisely what EFT predicts: the effective theory reproduces the true theory more closely as more short-distance physics is included.

Fig. (4) shows the results for the tensor polarization observable T_{20}. These values also include the counterterm mentioned above. Experimental data is shown with error bars, and the EFT calculations are well within for low momentum.



T20 with Qd counterterm
T20
Q (MeV)
0.0
−0.4
−0.8
-1.2
−1.6
−2.0
0.0
200.0
400.0
600.0
800.0
1000.0
Nijm93
R=2.03
R=1.5
R=2.5
R=3.5
Older data
NIKHEF 1998
JLab Preliminary

Figure 4: T_{20}

5 Future (and Current) Work

The calculation of form factors by "integrating in" has, so far, only been taken up to order δ in the currents, where δ is the ratio of a characteristic momentum to the nucleon mass. I am currently extending the calculation to order δ^2, which involves only $j_0^{(1)}(\mathbf{p}, \mathbf{Q})$ (the next correction to j_+ occurs at order δ^3). One of the primary motivations for this extension is that the charge radius of the neutron comes in at this order. This quantity can be extracted from the EFT results and compared with experiment. As the experimental value is difficult to obtain and a theoretical one usually inaccurate, a prediction of it might be a great success of EFT, which appears to suffer from none of the shortcomings of the other two approaches.

Acknowledgements

I would like to thank my advisor, Daniel Phillips, for introducing me to these interesting problems and teaching me a great deal this summer, the EFT group for providing a stimulating environment for doing physics, Wick Haxton and Martin Savage for organizing this excellent REU program, and the INT and the NSF for funding it.

References

[1] T. deForest and J.D. Walecka, Adv. Phys. **15**, 1 (1966).

[2] D.R. Phillips and T.D. Cohen, nucl-th/9906091.

[3] Francis Halzen and Alan D. Martin, *Quarks & Leptons: An Introductory Course in Modern Particle Physics* (Wiley, New York, 1984).

[4] G. P. Lepage, "How to Renormalize the Schrödinger Equation," nucl-th/9706029, 1997.