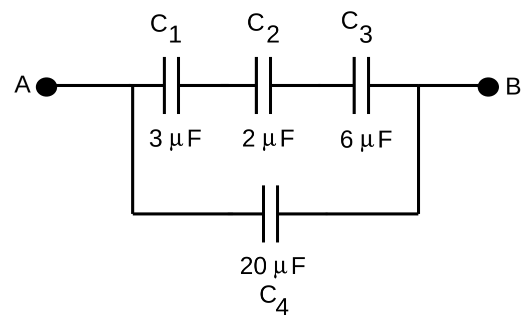
Name (Please Print) ______________________________ UW ID# _____ Score _____
(last) (first)

Capacitors
Four capacitors are arranged as shown in the figure. A potential difference of 40 V is applied between the points A and B, as shown in the figure.

Question 1 [5 points]
What is the potential difference across capacitor C_4?



(a) 0

(b) 2 V

(c) 20 V

(d) 40 V

(e) 60 V

Question 2 [5 points]
What is the magnitude of the excess charge on each plate of capacitor C_4?

(a) 0 μC

(b) 20 μC

(c) 40 μC

(d) 800 μC

(e) 500 nC

Question 3 [5 points]
What is the capacitance of the system?

(a) $\frac{220}{31}$ μF

(b) 21 μF

(c) $\frac{20}{21}$ μF

(d) 31 μF

(e) 720 μF

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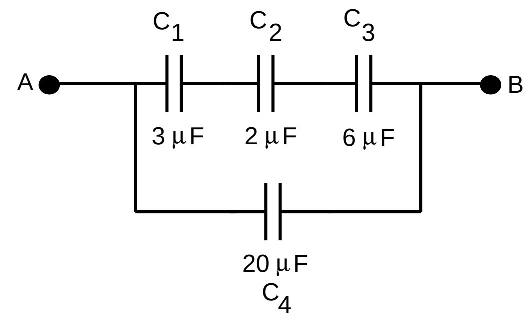
Question 4 [5 points]
What is the magnitude of the charge on each plate of capacitor C_2?



(a) 80 μC

(b) 20 μC

(c) 440 μC

(d) 84 μC

(e) 40 μC

Question 5 [5 points]
What is the potential difference across capacitor C_1?

(a) 13.3 V

(b) 20 V

(c) 40 V

(d) 2 V

(e) 6.6 V

Question 6 [5 points]
How much energy is stored in the system?

(a) 16.8 kJ

(b) 3.4 J

(c) 16.8 mJ

(d) 0.84 μJ

(e) 1 J

Question 7 [5 points]
Capacitor C_1 is an ideal parallel plate capacitor that is filled with material with a dielectric constant of $\kappa = 2$. What is the approximate area of each plate if the plate separation is 8.85×10^{-6} m?

(a) 8.85×10^{-6} m^2

(b) 4.42×10^{-6} m^2

(c) 3.0 m^2

(d) 0.75 m^2

(e) 1.5 m^2

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Current and Resistance
A battery producing a potential difference of 12 V is connected across an ohmic filament. A current of 3 A flows through the battery.

Question 8 [5 points]
What is the resistance of the filament?

(a) 48 Ω

(b) 12 Ω

(c) 4 Ω

(d) 3 Ω

(e) 1 Ω

Question 9 [5 points]
If the filament has a cross-sectional area of 0.2 mm^2 and a length of 1 cm, what is the current density inside the filament assuming that the current is uniformly distributed in the filament?

(a) 4 A/cm^2

(b) 0.8 A/mm^2

(c) 12 V

(d) 3×10^7 A/m^2

(e) 1.5×10^7 A/m^2

Question 10 [5 points]
What is the resistivity of the material that forms the filament?

(a) 3 Ω

(b) $2 \times 10^{+5}$ Ω m

(c) 8×10^{-9} Ω m^3

(d) 8×10^{-5} Ω m

(e) 3 A

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(last) (first)

Question 11 Potential Energy and Potential 20 points total

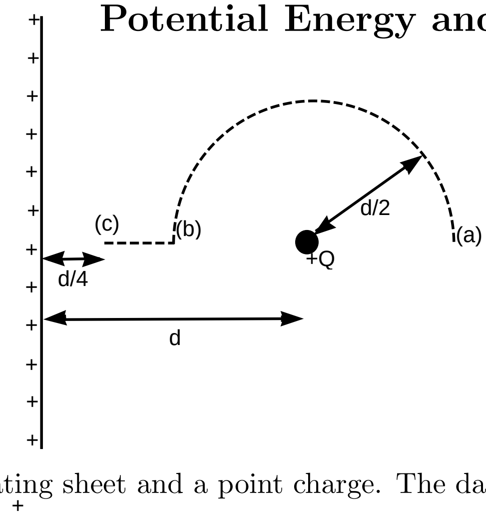

FIG. 1. A charged insulating sheet and a point charge. The dashed lines denote paths taken by a test charge.

An infinite, insulating sheet with surface charge density σ is located and fixed at $x = 0$. A point charge $+Q$ is located and fixed at $\mathbf{r} = d\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, as shown in figure 1.

A. [**5** points] What is the force experienced by the charge $+Q$ due to the infinite sheet of charge?

$$\mathbf{E}(x > 0) = \frac{\sigma}{2\epsilon_0}\,\hat{\mathbf{i}}$$
$$\mathbf{F}_Q = \frac{\sigma}{2\epsilon_0} Q\,\hat{\mathbf{i}} \quad .$$

B. [**5** points] How much work must be done to move a test charge q_t from the point (b), $\mathbf{r} = d/2\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$ to the point (c), $\mathbf{r} = d/4\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, along the straight-line path shown in figure 1?

Using the result from part (c), gives

$$w = -\Delta U \;=\; -q_t\left(\frac{Q}{6\pi\epsilon_0 d} \;-\; \frac{\sigma d}{8\epsilon_0}\right) \quad .$$

C. [**5** points] What is the magnitude of the potential difference between the points (b) and (c)?

The potential at any point receives contributions from both the point charge and the plane of charge density:

$$V_\sigma(x) = -\frac{\sigma}{2\epsilon_0}\,x \quad , \quad V_Q \;=\; \frac{Q}{4\pi\epsilon_0|\mathbf{d}-\mathbf{x}|} \quad .$$

Therefore,

$$V^{(b)} = \frac{Q}{2\pi\epsilon_0 d} - \frac{\sigma d}{4\epsilon_0} \quad , \quad V^{(c)} = \frac{Q}{3\pi\epsilon_0 d} - \frac{\sigma d}{8\epsilon_0}$$

$$V^{(b)} - V^{(c)} = \frac{Q}{6\pi\epsilon_0 d} - \frac{\sigma d}{8\epsilon_0} \quad .$$

D. [**5** points] How much work must be done to move a test-charge q_t from the point (a), $\mathbf{r} = 3d/2\ \hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$ to the point (b), $\mathbf{r} = d/2\ \hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$ along the semi-circular path shown in figure 1?

This is easy since the circular path is an equipotential of Q, we need only consider the contribution from the wall of charge.

$$\Delta V = -\frac{\sigma d}{2\epsilon_0} \quad , \quad w = +q_t \frac{\sigma d}{2\epsilon_0} \quad .$$

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Question 12 Capacitors and Dielectrics 10 points total

A parallel plate capacitor with plates of area A, located at $z = 0$ and $z = d$ is half filled with a material of dielectric constant κ, that extends from $z = 0$ to $z = d/2$. The plate at $z = d$ is at a potential of V_0, while the plate at $z = 0$ is at $V = 0$. Neglect fringing effects.

A. [**5** points] What is the potential at each point in the gap?

We start by using Gauss's law to determine the electric field in the gap. Using a "Pill-Box" shaped Gaussian surface the extends into a region with dielectric constant κ at one end, and in the conducting plate of the capacitor at the other.

$$\kappa\ \epsilon_0 \int d\mathbf{S}\cdot\mathbf{E} = Q_{\rm inc}$$
$$|\mathbf{E}| = \frac{\sigma}{\kappa\ \epsilon_0} \quad .$$

Therefore, if the capacitor is half filled with dielectric κ_1 (from $x = 0$ to $x = d/2$) and the half filled with dielectric κ_2, the potential difference between the plates is

$$\Delta V = -\int_0^d dx\ |\mathbf{E}(x) \ = \ \frac{d}{2}\left[\frac{\sigma}{\kappa_1\ \epsilon_0} + \frac{\sigma}{\kappa_2\ \epsilon_0}\right] \ = \ \frac{\sigma\ d}{2\ \epsilon_0}\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2} \ = \ V_0 \quad .$$

It now follows that the potential at each point inside the capacitor is

$$V(z < \frac{d}{2}) = \frac{\sigma}{\kappa_1\ \epsilon_0}\ z \ = \ V_0\ \frac{z}{d}\ \frac{2\kappa_2}{\kappa_1+\kappa_2}$$
$$V(z > \frac{d}{2}) = V_0\ \frac{\kappa_2}{\kappa_1+\kappa_2} \ + \ V_0\ \frac{(z-\frac{d}{2})}{d}\ \frac{2\kappa_1}{\kappa_1+\kappa_2} \quad .$$

B. [**5** points] What is the capacitance of the system?

From the previous result,

$$\frac{\sigma\ d}{2\ \epsilon_0}\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2} = V_0$$
$$\frac{Q\ d}{2\ \epsilon_0\ A}\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2} = V_0 \ = \ \frac{Q}{C}$$
$$C = \frac{\epsilon_0\ A}{d}\ \frac{2\kappa_1\kappa_2}{\kappa_1+\kappa_2} \quad .$$

The we can set $\kappa_1 = 1$ and $\kappa_2 = \kappa$ to recover the solution to the case considered.