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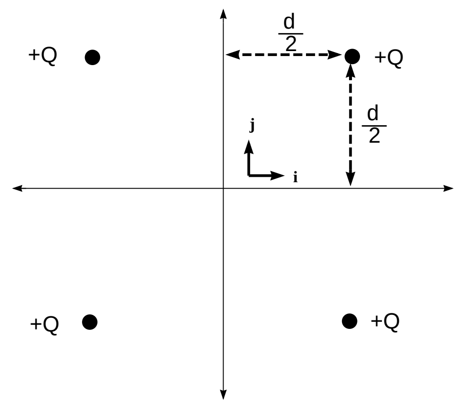
Electric Forces and Fields

Four equal charges are located on the corners of a square with sides of length d (in units of meters). Let $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ be unit vectors in the x- and y-directions, respectively.

Question 1 [5 points]

What is the force on the charge located at $\mathbf{r} = \frac{d}{2}\hat{\mathbf{i}} + \frac{d}{2}\hat{\mathbf{j}}$.



(a) 0

(b) $+\frac{KQ^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$

(c) $+\frac{KQ^2}{d^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$

(d) $-\frac{KQ^2}{d}\left(1+\frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$

(e) $+\frac{KQ^2}{d^2}\sqrt{\frac{5}{4}}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$

Question 2 [5 points]

What is the electric field at the center of the square?

(a) 0

(b) ∞

(c) $+\frac{4KQ}{d^2}$

(d) $+\frac{KQ}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$

(e) $+\frac{KQ}{d^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$

Question 3 [5 points]

What is the electric field at the point $\mathbf{r} = -\frac{d}{2}\hat{\mathbf{i}}$?

(a) 0

(b) $-\frac{2KQ}{d^2}\ \hat{\mathbf{i}}$

(c) $-\frac{2KQ}{d^2}\left(\frac{4}{5}\right)^{3/2}\ \hat{\mathbf{i}}$

(d) $-\frac{2KQ}{d^2}\left(\frac{8}{5}\right)\ \hat{\mathbf{i}}$

(e) $+\frac{2KQ}{d^2}\ \hat{\mathbf{j}}$

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Coulombs Law
Two identical point-like conducting spheres are placed at a distance of $d = 3$ m from each other. One sphere is charged with $q = 1$ nC while the other is charged with $q = -9\ \mu$C.

Question 4 [5 points]
What is the approximate magnitude of the force between them?

(a) 2×10^{-2} N

(b) 0 N

(c) 3×10^{-6} N

(d) 9×10^{-6} N

(e) 9×10^{-3} N

Question 5 [5 points]
What is the direction of the force between them?

(a) Perpendicular to the line between the centers of the sphere, attractive

(b) Along the line between the centers of the sphere, repulsive

(c) Along the line between the centers of the sphere, attractive

(d) Force has no direction

(e) Both (a) and (c)

The spheres are brought into contact with each other, and then returned to their original positions.

Question 6 [5 points]
What is the approximate magnitude of the force between them?

(a) 2×10^{-2} N

(b) 0 N

(c) 3×10^{-6} N

(d) 9×10^{-6} N

(e) 9×10^{-3} N

Question 7 [5 points]
What is the direction of the force between them?

(a) Perpendicular to the line between the centers of the sphere, attractive

(b) Along the line between the centers of the sphere, repulsive

(c) Along the line between the centers of the sphere, attractive

(d) Force has no direction

(e) Both (a) and (c)

Midterm 1, Physics 122B, Spring 2007, Savage

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Electric Dipoles
An electric dipole is formed by a charge $q_1 = -1\ \mu$C located at $z = -1$ mm along the z-axis and a charge $q_2 = +1\ \mu$C located at $z = +1$ mm along the z-axis, joined together by a rigid insulating rod.

Question 8 [5 points]
What is the dipole moment, $\mathbf{p}$, of this system? ($\hat{\mathbf{k}}$ is the unit vector in the $+z$-direction.)

(a) 1μC

(b) $-4 \times 10^{-9}\ \hat{\mathbf{k}}$ Cm

(c) $2 \times 10^{-9}\ \left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ Cm

(d) $-2 \times 10^{-9}\ \hat{\mathbf{k}}$ Cm

(e) $2 \times 10^{-9}\ \hat{\mathbf{k}}$ Cm

Question 9 [5 points]
What direction is the electric field at a distance of 10 m from the dipole in the x-y plane?

(a) radially outward from the dipole.

(b) radially inward toward the dipole

(c) in the +z direction

(d) in the -z direction

(e) the field vanishes

Question 10 [5 points]
A uniform electric field, $\mathbf{E} = E_0\ \left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right)$, is applied to the system, with $E_0 = 2 \times 10^{+3}$ N/C. What is the torque on the dipole.

(a) 4×10^{-6} Nm

(b) $-4 \times 10^{-6}\ \hat{\mathbf{i}}$ Nm

(c) $4 \times 10^{-6}\ \hat{\mathbf{j}}$ Nm

(d) $-2\sqrt{2} \times 10^{-9}\ \hat{\mathbf{i}}$ Nm

(e) 0

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Question 11 Gauss's Law 30 points total

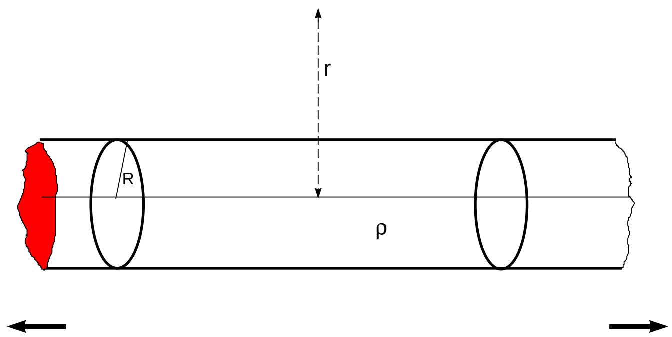

FIG. 1. A charged cylinder with charge per unit volume $\rho(r) = Ar^2$ for $r < R$.

Consider an infinitely long cylindrical charge distribution of radius R, with charge per unit volume $\rho(r) = A\, r^2\, C/m^3$ (where A is a constant), for $r < R$.

A. [**5** points] Draw a cylindrical Gaussian surface on fig. 1 that you can use to determine the electric field at a given distance $r > R$ from the axis of the charge distribution.

The Gaussian surface is a finite-length cylinder of radius r co-axial with the charge distribution, and of length L.

B. [**5** points] What is the charge enclosed, $Q_{\rm in}$, by the Gaussian surface you have shown in (a) ?

$$Q_{\rm in} = \int dV\ \rho(r) = \int_{L_0}^{L_0+L} dz \int_0^{2\pi} d\theta \int_0^R dr\ r\ Ar^2$$
$$= A\ L\ 2\pi \int_0^R dr\ r^3 = \frac{1}{2} AL\pi R^4$$

C. [**5** points] Use Gauss's law to find the electric field at a distance $r > R$ from the axis of the cylinder, in terms of $Q_{\rm in}$.

$$\int d\mathbf{S} \cdot \mathbf{E} = \frac{1}{\epsilon_0} \int dV\ \rho(\mathbf{r})$$
$$|\mathbf{E}| 2\pi r L = \frac{1}{\epsilon_0} Q_{\rm in}$$
$$|\mathbf{E}| = \frac{Q_{\rm in}}{2\pi\epsilon_0 r L} = \frac{AR^4}{4\epsilon_0 r}$$
$$\mathbf{E} = |\mathbf{E}|\hat{\mathbf{r}}$$

D. [**10** points] Find the electric field at a distance $r < R$ from the axis of the cylinder.

The Gaussian surface is now contained within the charge distribution, and

$$
\begin{aligned}
Q_{\rm in} = \int \ dV \ \rho(r) \quad &= \quad \int_{L_0}^{L_0+L} \ dz \ \int_0^{2\pi} \ d\theta \ \int_0^r \ dr' \ r' \ Ar'^2 \\
= A \ L \ 2\pi \ \int_0^r \ dr' \ r'^3 \quad &= \quad \frac{1}{2} AL\pi r^4
\end{aligned}
$$

which gives

$$
\begin{aligned}
\int \ d\mathbf{S} \cdot \mathbf{E} &= \frac{1}{\epsilon_0} \int \ dV \ \rho(\mathbf{r}) \\
|\mathbf{E}| 2\pi r L &= \frac{1}{\epsilon_0} Q_{\rm in} \\
|\mathbf{E}| &= \frac{Q_{\rm in}}{2\pi\epsilon_0 r L} \quad = \quad \frac{Ar^3}{4\epsilon_0} \\
\mathbf{E} &= |\mathbf{E}| \hat{\mathbf{r}}
\end{aligned}
$$

E. [**5** points] Sketch the magnitude of the electric field $|\mathbf{E}|$ between $r = 0$ and $r >> R$.

The magnitude of the electric field increase from zero at $r = 0$, *behaving as* $\propto r^3$ *until it reaches the outer edge of the charge distribution, and then it falls as* $1/r$ *for* $r > R$. *The field is continuous across* $r = R$.