Physics 572

Problem set #4 **Due Tuesday 5/24/2016**

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1. *3 points out of 10.* In class we discussed the $SU(3) \times SU(3)$ chiral lagrangian for low energy QCD. The term we wrote down to account for the electromagnetic splitting between the π^0 and the π^+ was

$$\mathcal{L}_{\rm em} = c\frac{\alpha}{4\pi} f_\pi^4 \mathrm{Tr}\, Q_L \Sigma Q_R \Sigma^\dagger \ , \qquad \Sigma = e^{2i\pi_a T_a/f_\pi} \tag{1}$$

where $f_\pi = 93$ MeV, and the $T_a \in SU(3)$, $a = 1, \ldots, 8$, with $\mathrm{Tr}\, T_a T_b = \frac{1}{2}\delta_{ab}$. The parameter c is an undetermined constant that has to be determined from data, and $Q_{L,R}$ are the left and right couplings of the photon:

$$Q_L = Q_R = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix} \tag{2}$$

Given the masses $m_\pi^0 = 135$ MeV, $m_\pi^\pm = 140$ MeV, compute the constant c. Recall that in the mostly minus convention, a free real scalar has the Lagrange density $\mathcal{L} = \frac{1}{2}\left[(\partial\phi)^2 - m^2\phi^2\right]$.

2. *7 points out of 10.* Consider a theory of three *ultraquarks* U, D, S which are triplets under a new $SU(3)$ strong interaction called *ultracolor*, which gets strong at a very high scale characterized by F_π, exactly analogous to f_π in QCD. The ultraquarks are assumed to be massless, and to condense the same way quarks do, but at the higher scale: the theory has an approximate $SU(3)_L \times SU(3)_R$ symmetry broken spontaneously to $SU(3)_V$. Now assume that the weak interactions $SU(2) \times U(1) \times U(1)'$ are gauged, where the $U(1)'$ is a new gauge symmetry with coupling constant g_0, while the $SU(2) \times U(1)$ are associated with the conventional Standard Model gauge group with couplings $g_{2,1}$. The ultraquarks are gauged with the following charges under $SU(2) \times U(1) \times U(1)'$:

$$\begin{pmatrix} U \\ D \end{pmatrix}_L = 2_{\frac{1}{6},\frac{1}{6}} \ , \qquad S_L = 1_{-\frac{1}{3},-\frac{1}{3}} \ , \qquad \begin{pmatrix} U \\ D \end{pmatrix}_R = 2_{\frac{1}{6},-\frac{1}{6}} \ , \qquad S_R = 1_{-\frac{1}{3},\frac{1}{3}} \ . \tag{3}$$

   (a) Consider the chiral Lagrangian for this theory, giving the conventional names π, K, η for the *ultramesons* of this theory. Assuming that $\langle\Sigma\rangle = \mathbf{1}$, what is the gauge symmetry breaking pattern and which of the eight Goldstone bosons get eaten?

   (b) Write down the terms $\mathcal{L}_{\rm gauged}$ analogous to the term in eq. (1) in Problem 1, but with the explicit contributions from all five of the $SU(2) \times U(1) \times U(1)'$ gauge generators. Discuss your answer with your classmates to make sure you get the right answer.

   (c) Use your expression for $\mathcal{L}_{\rm gauged}$ to compute the squares of the masses of the remaining pseudo Goldstone bosons in this theory. Do all of the gauge interactions make positive contributions to the square masses in this theory?

   (d) Consider vacua of the form

$$\Sigma = \exp\left[i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \theta \\ 0 & \theta & 0 \end{pmatrix} \right] \tag{4}$$

   and compute the potential $V(\theta)$ arising from $\mathcal{L}_{\rm gauged}$ and show that the minimum is at nonzero θ for $g_0 > g_c$, where you compute g_c (in terms of $g_{1,2}$).

(e) From the kinetic term

$$\mathcal{L}_{\text{kin}} = \frac{F_\pi^2}{4} \operatorname{Tr} D_\mu \Sigma D^\mu \Sigma^\dagger \tag{5}$$

compute the W, Z masses as functions of θ.

(f) Consider the limit $g_0 \to g_c$ from the $g > g_c$ side (equivalent to $0 < \theta \ll 1$), while $F_\pi \to \infty$ so that the W and Z bosons have their correct experimental values. In this limit, compute the Higgs mass$^2 = F_\pi^{-2} \partial_\theta^2 V(\theta)$ at the minimum of the potential. How does this compare to the experimental Higgs mass?