Physics 572

Problem set #1 **Due Tuesday 4/12/2016**

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1. The adjoint representation:

   (a) Show that for a Lie algebra $[T_a, T_b] = if_{abc}T_c$ with $\mathrm{Tr}\, T_a T_b = k\delta_{ab}$, then $[t_a, t_b] = if_{abc}t_c$ serves as a d-dimensional representation of the algebra, where d is the number of generators and the t_a are matrices defined as $(t_a)_{bc} = -if_{abc}$. Hint: consider the Jacobi identity.

   (b) Show that if I define $\Phi = \phi_a T_a$ then the two transformations $\delta_a \Phi = i\,[T_a, \Phi]$ and $\delta_a \phi_b = i(t_a)_{bc}\phi_c$ are equivalent.

2. (a) Starting from an $SU(5)$ GUT with three families of quarks and leptons broken to the SM at a scale $\mu = M_{\mathrm{GUT}}$, with α_5 equal to the unified fine structure constant at that scale, use the 1-loop β functions to scale the couplings $\alpha_{1,2,3}$ down to $\mu = M_Z$ (you can leave the top quark contribution in the β functions all the way down to M_Z). By using the experimentally determined values for $\alpha_s(M_Z)$ and $\alpha(M_Z)$, predict $\sin^2\theta_w$. How does your prediction compare with the experimental value? What do you find for α_5 and M_{GUT}?

   (b) Reverse the calculation: start with the experimental values for $\alpha_s(M_Z)$, $\alpha(M_Z)$ and $\sin^2\theta_w$ and plot c/α_1, $1/\alpha_2$, and $1/\alpha_3$ versus $\ln \mu/M_Z$, where c is the appropriate numerical factor indicated by $SU(5)$, arising from T_{24} and Y having different normalizations. What does your plot look like in the vicinity of μ equal to the value of M_{GUT} you found above? Can you think of how the assumptions could be changed to make the couplings all meet? (I only expect a qualitative answer for this last question)