Physics 571 – Winter 2018

Problem set #5 **Due Friday, 3/9/2018 (turn in to Natalie)**

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1. In QED with an electron but no other charged particles, with $\alpha(m_e) = \frac{1}{137}$, at what scale Λ does the 1-loop beta function predict that $\alpha(\Lambda) = \infty$? Express Λ in terms of the Planck mass, where quantum gravity gets strong, $m_P = \sqrt{\hbar c/G} = 10^{19}$ GeV.

2. Suppose you add to QED a new "dark photon" B_μ with field strength $B_{\mu\nu}$ which couples to a dark matter fermion χ with coupling g. The dark photon does not couple directly to the electron ψ, but has a small mixing term with the ordinary photon:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left[i(\partial\!\!\!/ + ieA\!\!\!/) - m\right]\psi + \bar{\chi}\left[i(\partial\!\!\!/ + igB\!\!\!/) - M\right]\chi \ . \qquad (1)$$

   Sho that there is a Coulomb force between the electron and the dark fermion. What is the relative strength to the ordinary Coulomb force? (Treat the ϵ term in perturbation theory as a $B - A$ vertex).