Physics 571 – Winter 2018

Problem set #4 **Due Thursday, 3/1/2018**

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1. Compute the differential cross section $\frac{d\sigma}{d\cos\theta}$ for $e^+e^- \to \mu^+\mu^-$ at tree level in QED in the limit $s \gg m_\mu^2 > m_e^2$, so that you can ignore the fermion masses.

2. Compute $\langle|\mathcal{T}|^2\rangle$ (averaged over initial spins, summed over final spins) for the process $e^+e^- \to e^+e^-$ (Bhabha scattering), at tree level in QED, in the limit that the electron is massless. Express your answer in terms of the Mandelstam variables s, t and u.

3. Consider the theory coupling a photon to a massless boson θ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \sqrt{2}efA_\mu\partial^\mu\theta - e^2f^2A_\mu A^\mu \ . \tag{1}$$

(a) Show that this theory has an exact gauge symmetry. (Hint: this is easy to see if you rewrite the theory in terms of $\Sigma = e^{i\theta/\sqrt{2}f}$.)

(b) Try to compute the photon propagator in this theory without gauge fixing.

(c) Add the gauge fixing term

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 \tag{2}$$

and compute the photon propagator in Landau gauge, $\xi = 0$. (Why is this a particularly simple gauge?) Where is the pole in the propagator?

(d) How might this Lagrangian might be generalized to a nonabelian $SU(N)$ gauge theory, where every gauge boson gets the same mass? Hint: try to construct the analogue of Σ. (Don't worry if you can't figure this out, it is difficult. But at the very least, figure out how many analogues of the θ field you would need).