Physics 571 – Winter 2018

Problem set #1 **Due Thursday, 1/11/2018**

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1. Large matrices can be built out of small matrices via direct products. For example, if a and b are n-dimensional matrices, and A, B are N-dimensional matrices, then $\mathcal{A} = a \otimes A$ and $\mathcal{B} = b \otimes B$ are each $(n \times N)$-dimensional matrices which can be multiplied together by the rule

$$\mathcal{A}\mathcal{B} = ab \otimes AB \ , \qquad \mathcal{B}\mathcal{A} = ba \otimes BA \ , \tag{1}$$

with other matrix operations defined as well, such as

$$\mathcal{A}^T = a^T \otimes A^T \ , \qquad \mathcal{A}^\dagger = a^\dagger \otimes A^\dagger \ , \qquad \mathrm{Tr}\,\mathcal{A} = \mathrm{Tr}\,a \times \mathrm{Tr}\,A \ , \qquad \det \mathcal{A} = (\det a)^N (\det A)^n \ . \tag{2}$$

   (a) Show that the following is a valid representation for the Dirac matrices (e.g., that they satisfy $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$ and $\{\gamma^\mu, \gamma_5\} = 0$):

$$\gamma^0 = \sigma_1 \otimes 1 \ , \qquad \gamma^j = i\sigma_2 \otimes \sigma_j \ , \qquad \gamma_5 = \sigma_3 \otimes 1 \ , \tag{3}$$

   where $j = 1, 2, 3$, the σ_j are the three Pauli matrices, and "1" is the 2-dimensional unit matrix.

   (b) Show in this basis that $(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$.

   (c) Find the matrix $\mathcal{C}$ in this basis that satisfies Eq. 40.46 in Srednicki (Hint: look for a solution which can be written as the direct product of two Pauli matrices and satisfies $\mathcal{C} = \mathcal{C}^{-1}$.)

2. For this problem, just give your results, do not show your work. Construct some basis for seven Dirac gamma matrices in seven spacetime dimensions out of direct products of Pauli matrices, satisfying $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$ where $\eta^{\mu\nu}$ is assumed to be diagonal with -1 in the time entry and $+1$ in the six space entries. Obviously there are many right answers. Can you also construct an analog of γ_5 which anticommutes with all seven of your γ^μ? How many components do Dirac spinors have in seven dimensions?

3. Use the fact that $\gamma_5^2 = 1$ and $\{\gamma_5, \gamma^\mu\} = 0$ to prove that the trace of an odd number of gamma matrices vanishes.