Problem set #7, Physics 571 **Due Monday 3/17/2014, 5PM (John Fuini's mailbox)**

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1. Compute the differential cross section $\frac{d\sigma}{d\cos\theta}$ for $e^+e^- \to \mu^+\mu^-$ at tree level in QED in the limit $s \gg m_\mu^2 > m_e^2$, so that you can ignore the fermion masses.

2. Compute $\langle|\mathcal{T}|^2\rangle$ for the process $e^+e^- \to e^+e^-$ (Bhabha scattering), at tree level in QED, in the limit that the electron is massless. Express your answer in terms of the Mandelstam variables s, t and u.

3. Consider the a theory coupling a photon to a massless boson θ:
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \sqrt{2}efA_\mu\partial^\mu\theta - e^2f^2A_\mu A^\mu \ . \tag{1}$$
   (a) Show that this theory has an exact gauge symmetry. (Hint: this is easy to see if you rewrite the theory in terms of $\Sigma = e^{i\theta/\sqrt{2f}}$.)
   (b) Try to compute the photon propagator in this theory without gauge fixing.
   (c) Add the gauge fixing term
$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 \tag{2}$$
   and compute the photon propagator in Landau gauge, $\xi = 0$. (Why is this a particularly simple gauge?) Where is the pole in the propagator?
   (d) How might this Lagrangian might be generalized to a nonabelian $SU(N)$ gauge theory, where every gauge boson gets the same mass? Hint: try to construct the analogue of Σ. (Don't worry if you can't figure this out, it is difficult. But at the very least, figure out how many analogues of the θ field you would need).

4. In this problem, we are going to take a simple gauge theory, free electromagnetism,
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ , \tag{3}$$
   and make it look complicated by a perverse choice of gauge. In class we used the Fadeev–Popov method to write the theory in "covariant gauge" that amounted to adding the gauge fixing term
$$\mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2 \ . \tag{4}$$
   The method was in principle capable of generating a ghost term, but in this case of a $U(1)$ gauge theory, it turned out that there were no ghosts.
   (a) If we follow the derivation in class, but replace the gauge fixing function $G = \partial_\mu A^\mu - h(x)$ (arbitrary $h(x)$) by
$$G = (\partial_\mu A^\mu + \beta A_\mu A^\mu) - h(x) \ , \tag{5}$$
   where β is a real number, we obtain
$$\mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu + \beta A_\mu A^\mu)^2$$
   plus ghost interactions. What are the ghost interactions? Give the Feynman rules for this theory.
   (b) The new theory looks like it has photon–photon scattering in tree approximation. Compute the relevant graphs and show that they sum to zero. (Note that ghosts don't enter the computation.)