1. Consider a Yukawa theory with a complex scalar field ϕ and a massless Dirac fermion Ψ:

$$\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi - \partial_\mu \phi^* \partial^\mu \phi - M^2 |\phi|^2 - g(\phi \bar{\Psi}_L \Psi_R + \phi^* \bar{\Psi}_R \Psi_L) - \frac{\lambda}{4} |\phi|^4 \ , \tag{1}$$

where $\Psi_L = P_- \Psi$, $\Psi_R = P_+ \Psi$, $\bar{\Psi}_L = \bar{\Psi} P_+$, $\bar{\Psi}_R = \bar{\Psi} P_-$, $P_\pm = (1 \pm \gamma_5)/2$.

(a) What are the symmetries of the theory? Explain why they imply that we do not need to include a counterterm for a Ψ mass, or scalar interactions such as ϕ^3.

(b) Write down the Feynman rules. Why would you want to put an arrow on the ϕ propagator?

(c) Draw the one-loop diagrams to be computed for $(Z_\Psi - 1)$, $(Z_\phi - 1)$, $(Z_M - 1)$, $(Z_g - 1)$ and $(Z_\lambda - 1)$ using the analogous definition of the Zs as was done for the Yukawa theory discussed in class and in Srednicki, chapter 51.

(d) Compute the coefficients of the $1/\epsilon$ poles for $(Z_\Psi - 1)$, $(Z_\phi - 1)$, $(Z_M - 1)$, $(Z_g - 1)$ and $(Z_\lambda - 1)$ at one loop, using dimensional regularization, where $d = 4 - \epsilon$ as in Srednicki.