Problem set #3, Physics 571 **Due Tuesday 1/28/2014**

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1. (a) Prove the identity $\det M = \exp \mathrm{Tr} \ln M$ for a finite-dimensional matrix M.

   (b) Use Srednicki eq 42.10 (which you proved last week in problem 5) to evaluate Tr $\ln(-\partial^2 + m^2 - i\epsilon)$ up to a constant (infinite) which is independent of m. How does this quantity relate to the result for the path integral for a free scalar field? Explain the physical significance of your result.

   (c) Do the the same for the fermionic quantity Tr $\ln(i\partial\!\!\!/ - m + i\epsilon)$.

   (d) Consider a theory as in problem (4d) in last week's problem set, for which the Lagrangian is

$$\mathcal{L} \;=\; \bar{\Psi} i\partial\!\!\!/ \Psi - \partial_\mu \Phi \partial^\mu \Phi^* - \mu^2 |\Phi|^2 + g\sqrt{2}\Phi\bar{\Psi}P_+\Psi + g\sqrt{2}\Phi^*\bar{\Psi}P_-\Psi \ , \tag{1}$$

   where $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$. After the Grassmann part of the integration has been performed we are left with a bosonic path integral of the form

$$\int D\Phi \, D\Phi^* \, e^{iS_\phi} \tag{2}$$

   where

$$S_\phi \equiv \int d^4x \left(-\partial_\mu \Phi \partial^\mu \Phi^* - \mu^2 |\Phi|^2\right) - \Delta(\Phi, \Phi^*) \ . \tag{3}$$

   What is the formal expression for Δ, arising from the Grassmann integration? Interpret the meaning of it.

   (e) Restricting to the case of a Φ field which is constant in spacetime, explain why $\Delta(\Phi, \Phi^*)$ must be a function that depends only on the absolute value of the field $|Phi|$, and is independent of the phase.