Physics 570 – Autumn 2017

Problem set #4 **Due Wednesday, 11/8/2017**

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1. Srednicki 11.2. Where the problem states to express the answer in terms of ω and ω', your answer will also depend on the electron mass m.

2. Consider the ϕ^4 theory in $d = 4$ dimensions described in Srednicki problem 28.1.

   (a) Draw all the divergent 1-loop diagrams in the theory and indicate which ones will contribute to the counterterms Z_m and Z_λ. Explain why none of these contribute to Z_ϕ, and draw all of the two-loop diagrams contributing to Z_ϕ.

   (b) Compute the Z_m and Z_λ counterterms at 1-loop order in the $\overline{\text{MS}}$ scheme. Note that it is easier to compute the counterterms than to compute the full result for these graphs!

   (c) Use your results and the methods of chapter 28 to compute the 1-loop anomalous dimension and β-function to $O(\lambda^2)$ for the theory.

   (d) Suppose that at $\mu = 1$ GeV experiments have determined $\lambda(\mu) = 1$. Plot $\lambda(\mu)$ for all μ. What happens in the far UV? In the far IR?

3. Suppose you have a theory with the RG equation

$$\frac{\partial g}{\partial \ln \mu} = \beta(g) = -c_1 g + c_2 g^3 \tag{1}$$

   where c_1 and c_2 are positive numbers. Assume that you can ignore higher order corrections to this equation.

   (a) Plot $\beta(g)$ versus g for all g. For $\beta > 0$, does g increase or decrease as μ is reduced (refer to the RG equation)? What about when $\beta < 0$ and $\beta = 0$? Indicate your answers by drawing arrows on your curve for how g flows in the IR (smaller μ).

   (b) Describe qualitatively the different behaviors this theory can have.

   (c) In general, why might it be difficult use perturbation theory to find a theory which reliably has a β function like this? Can you think of a condition on c_1 and c_2 that could make this possible?