Two of the following five problems will be graded and are worth five points each.

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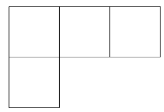
1. Georgi 12B

2. Consider the following representation of $SU(3)$:



   (a) What is the dimension of this representation?

   (b) Decompose this representation under the $SU(2) \times U(1)$ subgroup of $SU(3)$, as in §12.3 of Georgi.

   (c) Use your above result to plot the weight diagram for this representation.

3. Georgi 13B

4. The "index" of a representation R with generators $T_a^{(R)}$ (not necessarily reducible) is defined as

$$\mathrm{Tr}\, T_a^{(R)} T_b^{(R)} = k_R \delta_{ab} \ . \tag{1}$$

   (a) Prove the following 3 properties:

$$k_{\overline{R}} = k_R \tag{2a}$$
$$k_{R_1 \oplus R_2} = k_{R_1} + k_{R_2} \tag{2b}$$
$$k_{R_1 \otimes R_2} = d_1\, k_{R_2} + d_2 k_{R_1} \ , \tag{2c}$$

   where d_1 and d_2 are the dimensions of R_1 and R_2 respectively.

   (b) Using the above relations, and the fact that we conventionally normalize $k_{\square} = \frac{1}{2}$ for the defining representation of $SU(N)$, find $k_{\square\square}$ and $k_{\begin{smallmatrix}\square\square\\ \square\end{smallmatrix}}$ for $SU(3)$.

   (c) Recompute $k_{\square\square}$ for $SU(3)$, only this time by first decomposing the $\square\square$ representation under $SU(2) \times U(1)$, taking care to normalize the $U(1)$ charges correctly, and then computing $\mathrm{Tr}\, T_8^{\square\square} T_8^{\square\square}$.

5. (a) Show that for any irreducible representation,

$$\sum_a (T_a)^i_j (T_a)^j_k = C_2(R) \delta^i_k \ , \tag{3}$$

   where $C_2(R)$ is a constant called the Casimir of the representation.

   (b) Show that $C_2(R) d_R = k_R d_{\mathrm{adjoint}}$

   (c) Compute the Casimir for the $SU(3)$ representations $\square\square$ and $\begin{smallmatrix}\square\square\\ \square\end{smallmatrix}$.