Physics 325, Winter 2002 **Problem set #6** Due: Fri. 3/1/02

Assigned problems. *Two problems will be graded, and will be worth 50 points each.*

1. (a) As discussed in class, for a spin $s = 1/2$ electron with orbital angular momentum you can express the $|s, \ell, j, m\rangle$ states in terms of the $|s, m_s; \ell, m_\ell\rangle$ states. (Here $s = 1/2$, ℓ is the orbital angular momentum, j is the total angular momentum, and m_s, m_ℓ and m are the eigenvalues (in units of $\hbar$) of S_z, L_z and J_z respectively.) Do so for $s = 1/2$, $\ell = 2$ and the three cases: $\{j, m\} = \{5/2, 5/2\}$, $\{j, m\} = \{5/2, 3/2\}$, and $\{j, m\} = \{3/2, 3/2\}$. See the examples for $\ell = 0$ and $\ell = 1$ in the lecture notes.

(b) If $s = 1/2$, $\ell = 2$, $j = 3/2$ and $m = 3/2$ what is the expectation value for the z-component of the electron spin, $\langle \frac{1}{2}, 2, \frac{3}{2}, \frac{3}{2}|\hat{S}_z|\frac{1}{2}, 2, \frac{3}{2}, \frac{3}{2}\rangle$?

2. Imagine that the Hamiltonian for the hydrogen atom was modified to be

$$H = -\frac{\hbar^2}{2\mu}\vec{\nabla}^2 - \frac{e^2}{r} + \xi\vec{L}\cdot\vec{S} \ , \tag{1}$$

where ξ is a constant with dimensions of energy/$\hbar^2$, and $\vec{S}$ is the electron spin operator. Give the energies, degeneracies, and quantum numbers of the four lowest energy levels.

3. Consider two electrons confined to a wire of length a; the spatial wavefunction $\psi(x_1, x_2)$ obeys the 1-dimensional Schrodinger equation with the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\left(\frac{d}{dx_1^2} + \frac{d}{dx_2^2}\right) + V(x_1) + V(x_2) \ , \qquad V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases} \ . \tag{2}$$

(a) Verify explicitly that $\psi_{n_1,n_2}(x_1, x_2) \equiv \phi_{n_1}(x_1)\phi_{n_2}(x_2)$ is an energy eigenstate and find its energy, where

$$\phi_n(x) \equiv \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} \ . \tag{3}$$

(b) Show that

$$\int_0^a \int_0^a dx_1\, dx_2\, |\psi_{n_1,n_2}(x_1, x_2)|^2 = 1 \ .$$

How would you interpret $|\psi_{n_1,n_2}(x_1, x_2)|^2$?

(c) The full wavefunction for the two electrons (spin $s_1 = s_2 = 1/2$) will have both a spatial part and a spin part. Consider states of the form $|n_1,\, n_2,\, s,\, m_s\rangle$, where $n_{1,2}$ refers to electron numbers 1 and 2 being in the spatial wavefunctions $\phi_{n_1}(x_1)$ and $\phi_{n_2}(x_2)$ respectively, with total spin s for the pair, and total spin in the z direction given by m_s. Given that two electrons must have a wavefunction which is antisymmetric under interchange of the two particles, find the energies, and quantum numbers of the three lowest energy eigenstates available to these electrons.

Hint: You should find the linear combinations of the $|n_1,\, n_2,\, s,\, m_s\rangle$ states which are still eigenstates of H, S^2, S_z, and which are antisymmetric under interchange of the two particles.

(d) *To do this problem you have to do some integrals which are time consuming by hand, but which just take moments using Mathematica.* Compare the mean square separation between the two electrons, namely the expectation value $\langle(\hat{x}_1 - \hat{x}_2)^2\rangle$, for the two cases: (i) when the electrons are in the lowest possible energy state with z component of total spin $m = 0$; and (ii) when the electrons are in the lowest possible energy state with z component of total spin $m = 1$.

Your answers ought to be numbers time a^2. Can you interpret the relative size of this expectation value in the two cases?