Physics 325, Winter 2002 **Problem set #5** Due: Fri. 2/22/02

Assigned problems. *Two problems will be graded, and will be worth 50 points each.*

1. A spin 1/2 particle is measured to have $S_z = +\frac{\hbar}{2}$. Subsequently you measure the spin in the direction $\hat{n} = (0, \sin\theta, \cos\theta)$. Assume there are no magnetic fields in the apparatus that could cause the spin to precess between measurements.

(a) What is the probability that you will measure the electron spin in the $\hat{n}$ direction to equal $-\frac{\hbar}{2}$? Sketch your answer as a function of the angle θ.

(b) Explain why your answer makes sense for the three particular values $\theta = 0,\ \pi/2,\ \pi$.

2. Show that for a particle with both spin and orbital angular momentum, the $|s, \ell, j, m\rangle$ state is an eigenstate of the operator $\vec{L}\cdot\vec{S}$, and compute its eigenvalue. Remember that this state is a simultaneous eigenstate of the operators $\vec{S}^2$, $\vec{L}^2$, $\vec{J}^2$ and J_z with eigenvalues $\hbar^2 s(s+1)$, $\hbar^2\ell(\ell+1)$, $\hbar^2 j(j+1)$ and $\hbar m$ respectively. The total spin $\vec{J}$ is defined as $\vec{J} = (\vec{L} + \vec{S})$. **Hint**: *Rewrite $\vec{L}\cdot\vec{S}$ in terms of $\vec{S}^2$, $\vec{L}^2$ and $\vec{J}^2$.*

3. Particle 1 and particle 2 have spins $s_1 = s_2 = 1/2$. They have a spin-spin interaction (like you would expect for two little magnets) corresponding to the Hamiltonian:

$$H = c\,\vec{S}_1\cdot\vec{S}_2 \tag{1}$$

where $\vec{S}_1$ is the spin of particle 1, and $\vec{S}_2$ is the spin of particle 2.

(a) What is the sign of c if the spins preferentially anti-align?

(b) Use the Hamiltonian eq. (1) to compute the four energy eigenvalues for the two particles. (As in the hint for problem 1, rewrite $\vec{S}_1\cdot\vec{S}_2$ in terms of $\vec{S}_1^2$, $\vec{S}_2^2$ and $\vec{S}^2$, where $\vec{S}$ is the total spin, $\vec{S} = (\vec{S}_1 + \vec{S}_2)$).

(c) Identify the four different energy levels of H with the appropriate $|s_1, s_2, s, m_s\rangle$ state, where $s_1 = s_2 = 1/2$, and s and m_s refer to the total spin and the z-component of the total spin respectively.

(d) Identify the same four energy levels of H with the appropriate combinations of $|s_1, m_1, s_2, m_2\rangle$ states, where $s_1 = s_2 = 1/2$, and m_1 and m_2 are the eigenvalues (in units of $\hbar$) of the operators $(S_1)_z$ and $(S_2)_z$ respectively.