Assigned problems. *Two problems will be graded, and will be worth 50 points each.* Start this problem set early! The lecture notes on Landau levels will be posted Friday.

1. What are the energy splittings between Landau levels (in eV) for an electron confined to the $x-y$ plane, in the presence of a constant magnetic field of 10^4 Gauss in the z direction? Use the value for the "Bohr magneton", $\mu_B \equiv e\hbar/(2m_e c) = 0.6\times10^{-14}$ MeV G^{-1}, where "G" = Gauss.

2. Consider the quantities $\hat{z} = (\hat{x}+i\hat{y})$, $\hat{\bar{z}} = (\hat{x}-i\hat{y})$, $\hat{p}_z = (\hat{p}_x - i\hat{p}_y)/2$ and $\hat{p}_{\bar{z}} = (\hat{p}_x + i\hat{p}_y)/2$.

(a) Compute all the commutators of these four quantities, and show that $p_z = -i\hbar\frac{\partial}{\partial z}$, $p_{\bar{z}} = -i\hbar\frac{\partial}{\partial \bar{z}}$.

(b) Express the angular momentum operator $\hat{L}_z = (\hat{x}\hat{p}_y - \hat{y}\hat{p}_x)$ in terms of $\hat{z}$, $\hat{\bar{z}}$, $\hat{p}_z$, and $\hat{p}_{\bar{z}}$.

(c) We saw that for a particle of mass μ and positive charge q, the lowest Landau level wave functions took the form $\psi_0 = f(z)e^{-(\mu\omega_c/4\hbar)\bar{z}z}$, where $\omega_c = qB/(\mu c)$ is the cyclotron frequency and $f(z)$ is an arbitrary holomorphic function (that is, a function of z, but not $\bar{z}$. Show that the wave function

$$\psi_{0,m} \equiv N_m z^m e^{-(\mu\omega_c/4\hbar)\bar{z}z} \tag{1}$$

where m is an integer, is an eigenstate of angular momentum, with

$$\hat{L}_z \psi_{0,m} = \hbar m\, \psi_{0,m} \ .$$

3. Find the normalization constant N_m for the wavefunction $\psi_{0,m}$ in eq. (1), so that

$$\int dx\, dy\, |\psi_{0,m}|^2 = 1 \ .$$

Hint: do the 2 dimensional integral in polar coordinates r, θ, where $x = r\cos\theta$ and $y = r\sin\theta$. Note that $z = re^{i\theta}$ and $\bar{z} = re^{-i\theta}$.

4. By acting on the lowest Landau level wavefunctions $\psi_{0,m}$ with the raising operator $\hat{A}^\dagger$ discussed in lecture, find solutions to the Schrödinger equation with energy $\frac{3}{2}\hbar\omega_c$. What is the angular momentum L_z of the state $\hat{A}^\dagger\psi_{0,m}$?