Physics 325, Winter 2002 **Problem set #2** Due: Fri. 1/25/02

Assigned problems. *Both are to be turned and will be graded, worth 50 points each.*

1. A particle of mass m and charge q interacts with an electromagnetic field. The Hamiltonian is

$$H = \frac{(\vec{p} - \frac{q}{c}\vec{A})^2}{2m} + q\phi \ ,$$

where $\vec{A}(\vec{r}, t)$ is the vector potential and $\phi(\vec{r}, t)$ is the scalar potential (note the sign error in the $e\phi$ term in Gasiorowicz, eq. 13-19, where $q = -e$.).

(a) Consider the special case

$$\phi(\vec{r}) = -\mathcal{E} z \ , \qquad \vec{A} = 0 \ ,$$

where $\mathcal{E}$ is some constant. Show that the electromagnetic fields are $\vec{E} = \mathcal{E}\hat{z}$, $\vec{B} = 0$.

(b) Show that in any quantum state $|\psi, t\rangle$, the following quantum version of the Lorentz force law (appropriate for $\vec{B} = 0$) holds:

$$m\vec{a} = q\vec{E} \ ,$$

where

$$\vec{a} \equiv \frac{d^2}{dt^2}\langle\psi, t|\vec{r}|\psi, t\rangle \ .$$

Hint: review from Physics 324 the problem #2 on problem set #5 and its solution (see the Physics 324 assignments web page).

2. Now consider the special case

$$\phi(\vec{r}) = 0 \ , \qquad \vec{A} = (0, \, x\mathcal{B}, \, 0) \ ,$$

where $\mathcal{B}$ is some constant.

(a) Show that the electromagnetic fields are $\vec{E} = 0$ and $\vec{B} = \mathcal{B}\hat{z}$.

(b) Show that in any quantum state $|\psi, t\rangle$, the following quantum version of the Lorentz force law (appropriate for $\vec{E} = 0$) holds:

$$m\vec{a} = \frac{q}{c}\vec{v} \times \vec{B} ,$$

where

$$\vec{a} \equiv \frac{d^2}{dt^2}\langle\psi, t|\vec{r}|\psi, t\rangle , \qquad \vec{v} \equiv \frac{d}{dt}\langle\psi, t|\vec{r}|\psi, t\rangle .$$

If you are feeling strong, you might try to prove the Lorentz force law for arbitrary, space dependent (but time independent) electromagnetic fields (not required or expected!).