Problems chosen to be graded are marked by $\star$

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(1) *Gasiorowics 1-2*

Integrating eq. (1-1) over λ gives $U(T) = \frac{4}{c}E(T)$. Combining with eq. (1-12a) then yields $E(T) = \frac{ca}{4}T^4 \equiv \sigma T^4$. Plugging in numbers, $\sigma = 5.67 \times 10^{-5}$ erg/cm^2 sec K^4. Note that the book gives the wrong value for σ!

The total energy emmitted per sec by the sun is $4\pi R_\odot^2 E(T)$; at a distance r from the sun, this energy passes through a sphere of area $4\pi r^2$, so that the power per unit area at a distance r is $p = E(T)(R_\odot/r)^2 = \sigma T^4 (r/R_\odot)^2$. Therefore, with $r = d_\odot$, the power received on Earth is $p = \sigma T^4 (r_\odot/d_\odot)^2 = 1.4 \times 10^6$ erg/cm^2 sec. From this, with $R_\odot = 7 \times 10^{10}$ cm, $d_\odot = 1.5 \times 10^{13}$ cm, we get $T = [p(d_\odot/r_\odot)^2/\sigma]^{1/4} = 5800$ K.

$\star$ **(2)** *Gasiorowics 1-5*

From eq. (1-16) we have $E = h\nu - W = hc/\lambda - W$, where E is the electron energy. We have 2 measurements, $E_{1,2}$ and $\lambda_{1,2}$:

$$E_1 = 2.3 \text{ eV},\ \lambda_1 = 2.0 \times 10^{-5} \text{ cm}, \quad E_2 = 0.90 \text{ eV},\ \lambda_2 = 2.58 \times 10^{-5} \text{ cm} \ . \tag{1}$$

Our equation reads

$$E_1 = hc/\lambda_1 - W \ , \qquad E_2 = hc/\lambda_2 - W \ . \tag{2}$$

We can eliminate W by subtracting these two equations from each other, deriving

$$\begin{aligned} h = \frac{\lambda_1\lambda_2(E_2 - E_1)}{c(\lambda_1 - \lambda_2)} &= 4.15 \times 10^{-15}\,\text{eV}\cdot\text{sec} \\ &= 4.15 \times 10^{-21}\,\text{MeV}\cdot\text{sec} \\ &= 6.64 \times 10^{-27}\,\text{erg}\cdot\text{sec} \ . \end{aligned} \tag{3}$$

and we then get

$$W = hc/\lambda_1 - E_1 = 3.9 \text{ eV} \tag{4}$$

⋆(3) *Gasiorowics 1-6*

Use eq. (1-25) and $\lambda = c/\nu = hc/E$ (Note: the equation above 1-25 is wrong!...check the dimensions!). So

$$\left(\frac{1}{E'} - \frac{1}{E}\right) = \frac{1}{mc^2}(1 - \cos\theta) \tag{5}$$

where E and E' are the initial and final photon energies, and $m = 938\ \mathrm{MeV}/c^2$ is the proton mass. To maximize the photon's energy loss, we must minimize E', which means we must maximize the right hand side of the above equation; this occurs for $\cos\theta = -1$ (180 degree scattering). In this case

$$\left(\frac{1}{E'} - \frac{1}{E}\right) = \frac{2}{mc^2} \tag{6}$$

Plugging in $E = 100$ MeV yields $E' = 82$ MeV, so the maximum photon energy loss is 18 MeV.

(4) *Gasiorowics 1-12*

$E = p^2/(2m)$, $p = h/\lambda$ so

$$E = h^2/(2m\lambda^2) = \frac{1}{2}mc^2((h/mc)/\lambda)^2 = \frac{1}{2}mc^2(\lambda_C/\lambda)^2 \ , \tag{7}$$

where $\lambda_C = 2.4 \times 10^{-10}$ cm is the Compton wavelength of the electron (eq 1-26), and $mc^2 = 0.51$ MeV.

Plugging in $\lambda = 150$ Angstrom $= 1.5\times10^{-6}$ cm, I get $E = 6.7\times10^{-3}$ eV; for $\lambda = 5.0 \times 10^{-8}$ cm, $E = 6.0$ eV.

⋆(5) *Gasiorowics 1-15*

For circular orbits, $mv^2/r = -F = dV/dr$ where F is the central force in the radial direction, $V(r)$ is the potential. Here $V = m\omega^2r^2/2$ so $F = -m\omega^2 r$ and therefore we find $v^2 = \omega^2r^2$, or $v/r = \omega$. This means that the classical frequency of the orbit is the parameter ω in the potential.

The energy is $E = mv^2/2 + m\omega^2 r^2/2 = m\omega^2 r^2$, given the above result. The angular momentum is $L = mvr = m\omega r^2$ applying the Bohr quantization condition $L = n\hbar$ we get $m\omega r^2 = n\hbar$ and so the energy of the allowed orbits is $E_n = n\hbar\omega$.

To check the correspondence principle, consider the frequency of light emitted in a transition from the $(N+1)$ orbit to the N orbit: $E_\gamma = \hbar\omega_\gamma = E_{N+1} - E_N \hbar\omega$, from which it follows (for any N) that the photon frequence ω_γ equals the electron's orbital frequency, ω. Thsi is the classical result, and so the correspondence principle is satisfied; that it is satisfied for any N is peculiar to the harmonic oscillator potential.

⋆ **(6)** We want to find a differential equation that admits plane waves $\psi = e^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar}$, but only with the relativistic dispersion relation $E = \sqrt{p^2c^2 + m^2c^4}$. We see that $-i\hbar\partial_t\psi = E\psi$ and $-\hbar^2\nabla^2\psi = p^2\psi$. Therefore one might be tempted to write the equation

$$-i\hbar\partial_t\psi = \left(\sqrt{-c^2\hbar^2\nabla^2 + m^2c^4}\right)\psi \ , \tag{8}$$

but this isn't a good idea since the square root of the differential operator causes problems. So try squaring it:

$$-\hbar^2\partial_t^2\psi = \left(-c^2\hbar^2\nabla^2 + m^2c^4\right)\psi \ . \tag{9}$$

This is a fine equation. However if we tried plugging in our plane wave guess, we would find it to be a solution so long as $E^2 = (p^2c^2 + m^2c^4)$, and equation with two solutions for the energy,

$$E = \pm\sqrt{p^2c^2 + m^2c^4} \ . \tag{10}$$

The negative solution is particularly worrisome...what is the meaning of negative energy? Why doesn't the hydrogen keep decaying to more and more negative energy states? Schrödinger couldn't answer the question, but Dirac could. He interpreted the negative energy electron states as positive energy positron states, where the positron is an anti-electron. His prediction that quantum mechanics plus relativity implies the existence of antimatter was soon confirmed with the discovery of the positron.