Assigned problems. *All three are to be turned in; two will be graded and will be worth 50 points each.*

1. **a)** Show that if $\hat{A}$, $\hat{B}$ and $\hat{C}$ are operators, then in general

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B} \ .$$

b) Given that $[\hat{x}, \hat{p}] = i\hbar$, compute $[\hat{x}, \hat{p}^n]$ and $[\hat{p}, \hat{x}^n]$.

c) Show that if $V(x)$ is a function with a convergent Taylor expansion, then $[\hat{p}, V(\hat{x})] = -i\hbar\, dV(\hat{x})/d\hat{x}$.

d) For the simple harmonic oscillator ladder operators $\hat{a}$ and $\hat{a}^\dagger$, where $[\hat{a}, \hat{a}^\dagger] = 1$, compute both $[\hat{a}^\dagger\hat{a}, \hat{a}]$ and $[\hat{a}^\dagger\hat{a}, \hat{a}^\dagger]$.

e) If $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ are the three angular momentum operators in three dimensions. They satisfy the commutation relations

$$[\hat{L}_x, \hat{L}_y] = i\hat{L}_z \ , \qquad [\hat{L}_y, \hat{L}_z] = i\hat{L}_x \ , \qquad [\hat{L}_z, \hat{L}_x] = i\hat{L}_y \ . \tag{1}$$

Compute the commutator $[\hat{L}_z, \hat{L}^2]$, where $\hat{L}^2 \equiv \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$.

2. **a)** Show that if a quantum particle is in state $|\psi, t\rangle$ which is a solution to the time dependent Schrodinger equation

$$i\hbar\frac{d}{dt}|\psi, t\rangle = \hat{H}|\psi, t\rangle \ , \tag{2}$$

then the expectation value of any operator $\hat{O}$ satisfies:

$$\frac{d}{dt}\langle\psi, t|\hat{O}|\psi, t\rangle = \frac{1}{i\hbar}\langle\psi, t|[\hat{O}, \hat{H}]|\psi, t\rangle \tag{3}$$

b) A *classical* particle moving in a potential $V(x)$ obeys the following relations:

$$\frac{dx}{dt} = \frac{p}{m} \ , \qquad \frac{dp}{dt} = -\frac{dV(x)}{dx} \ .$$

Using eq. (3) above and the results from problem (1), show that a quantum particle with Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

satisfies the analogous relations:

$$\frac{d}{dt}\langle\psi, t|\hat{x}|\psi, t\rangle = \frac{1}{m}\langle\psi, t|\hat{p}|\psi, t\rangle \ , \tag{4}$$

$$\frac{d}{dt}\langle\psi, t|\hat{p}|\psi, t\rangle = -\langle\psi, t|\frac{dV(\hat{x})}{d\hat{x}}|\psi, t\rangle \ . \tag{5}$$

3. Consider a particle of mass m in a one dimensional harmonic oscillator potential, $V(x) = \frac{1}{2}kx^2$. Suppose that at time $t = 0$ its wavefunction is given by

$$|\psi, 0\rangle = N\left(|3\rangle - i|4\rangle\right) \ ,$$

where $|n\rangle$ are the orthonormal harmonic oscillator energy eigenstates which we discussed in class.

a) Compute N so that the state is normalized.

b) Compute the expectation value of the position, as a function of time

$$\langle\psi, t|\hat{x}|\psi, t\rangle \ .$$

Hint: You do not need to know the wave functions $u_3(x)$ and $u_4(x)$ or to compute an integral to solve this problem. Express $\hat{x}$ in terms of $\hat{a}$ and $\hat{a}^\dagger$.

c) Compute the expectation value of the momentum, as a function of time

$$\langle\psi, t|\hat{p}|\psi, t\rangle \ .$$

Do your results satisfy both eq. (4) and eq. (5)?