Physics 324, Fall 2001 **Problem set #4** Due: Fri. 11/9/01

Assigned problems. *All four are to be turned in; problem (3) and one other will be graded and will be worth 50 points each.*

1. Gasiorowicz, Ch. 4, #4

2. Gasiorowicz, Ch. 4, #6

⋆ **3.** Consider a particle confined to a one dimensional box of size a, as discussed in Ch. 4. Suppose at time $t = 0$ the wave function is

$$|\psi, t = 0\rangle = N\left(2|1\rangle + |2\rangle\right) \ , \tag{1}$$

where $|n\rangle$, $n = 1, 2, 3 \ldots$, are the energy eigenstates of the time independent Schrodinger equation:

$$\hat{H}|n\rangle = E_n|n\rangle = \left(\frac{(\hbar n\pi/a)^2}{2m}\right)|n\rangle \ . \tag{2}$$

Note that if we denote the position eigenstates to be $|x\rangle$, representing a wavefunction where the particle is localized exactly at position x, then

$$\langle x|n\rangle = u_n(x) \ , \quad \langle n|x\rangle = u_n^*(x) \ , \quad u_n(x) = u_n^*(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases} \ . \tag{3}$$

The orthonormality and completeness relations are

$$\langle m|n\rangle = \delta_{mn} \ , \qquad \sum_{n=1}^{\infty} |n\rangle\langle n| = 1 \ . \tag{4}$$

a) Use the bra ket language to compute the coefficient N which normalizes $|\psi, t = 0\rangle$:

$$\langle \psi, t = 0|\psi, t = 0\rangle = 1 \ . \tag{5}$$

b) Show that

$$|\psi, t\rangle \equiv N\left(2e^{-iE_1 t/\hbar}|1\rangle + e^{-iE_2 t/\hbar}|2\rangle\right) \tag{6}$$

is a solution to the time dependent Schrödinger equation,

$$i\hbar\frac{d}{dt}|\psi, t\rangle = \hat{H}|\psi, t\rangle \tag{7}$$

with the initial condition for $|\psi, t = 0\rangle$ given in eq.(1) above.

c) Make the connection with more conventional language: what is $\psi(x,t) \equiv \langle x|\psi,t\rangle$, given the above solution for $|\psi,t\rangle$?

d) Compute the expectation values for the energy for this solution:

$$\langle E\rangle = \langle \psi,t|\hat{H}|\psi,t\rangle \ . \tag{8}$$

Does $\langle E\rangle$ depend on time? (If it did, that would be very strange, since it would imply violation of energy conservation!). Can you interpret your result?

e) Compute the expectation value for the particle position for this solution $\langle x\rangle = \langle \psi,t|\hat{x}|\psi,t\rangle$, and sketch it as a function of time.

Hint: you should use the properties of the position eigenstates,

$$\hat{x}|x\rangle = x|x\rangle \ , \quad \langle x|x'\rangle = \delta(x-x') \ , \quad \int_{-\infty}^{\infty} dx\,|x\rangle\langle x| = \hat{1} \ , \tag{9}$$

and use the fact that you can insert the unit operator $\hat{1}$ between operators or states without changing anything. For example, suppose you needed to compute $\langle m|\hat{x}|n\rangle$:

$$\begin{aligned}\langle m|\hat{x}|n\rangle = \int_{-\infty}^{\infty} dx \langle m|\hat{x}|x\rangle\langle x|n\rangle &= \int_{-\infty}^{\infty} dx\, x\, \langle m|x\rangle\langle x|n\rangle \\ &= \int_{-\infty}^{\infty} dx\, x\, u_m^*(x)u_n(x) \\ &= \frac{2}{a}\int_0^a dx\, x\, \sin\frac{m\pi x}{a}\sin\frac{n\pi x}{a} \ . \end{aligned} \tag{10}$$

4. Gasiorowicz, Ch. 6, #5

Hint: Use the completeness of the energy eigenfunctions, $\sum_{n=1}^{\infty}|n\rangle\langle n| = \hat{1}$, and the orthonormality of the position eigenfunctions, $\langle x|x'\rangle = \delta(x-x')$.