Physics 324, Fall 2001 **Problem set #2** Due: Fri. 10/19/01

Assigned problems. *All four are to be turned in; two will be graded and will be worth 50 points each.*

1. **a)** Consider a particle of mass m confined to a very thin wire of length L; you can treat this as a 1-dimensional problem. Use the uncertainty relation $\Delta p \Delta x \gtrsim \hbar$ to estimate the lowest energy ("ground state energy") that this particle can have, in terms of m, L, and $\hbar$.

b) Roughly what is the minimum velocity that the particle can have when confined to this wire (in terms of m, L, and $\hbar$)?

c) Compute the numerical value of the velocity found above (in cm/sec) for two cases: (i) The particle is an electron, $m = 0.5 MeV/c^2$, and $L = 1$ nm ("nm" means "nanometer"$=10^{-9}$ meter); (ii) The particle is a pea ($m = 1$ gm) in a drinking straw ($L = 10$ cm).

2. **a)** A nonrelativistic particle of mass m is in a 1-dimensional potential $V(x) = V_0(x/a)^4$, where V_0 is a constant with dimensions of energy, and a is a constant with dimensions of length. Sketch $V(x)$. Use the uncertaintly relation to estimate the groundstate energy of the particle in this potential.

b) For what ranges of a can one no longer assume that the particle is nonrelativistic? Express your answer in terms of the particle's Compton wavelength.

3. **a)** Consider a wave packet with the momentum space wave function $\phi(p) = Ne^{-(p-\bar{p})^2/2p_0^2}$. Normalize $\phi(p)$ and find N (e.g., so that $\int_{-\infty}^{\infty} dp\, |\phi(p)|^2 = 1$).

b) $|\phi(p)|^2$ is interpreted as the probability density for finding the particle to have momentum p. Thus the average value for a measurement of p^n is $\langle p^n \rangle = \int_{-\infty}^{\infty} dp\, p^n |\phi(p)|^2$. Compute the average momentum $\langle p \rangle$ for a particle in this wave packet, the average momentum squared $\langle p^2 \rangle$, and the root mean square momentum, $\Delta p \equiv \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.

Problem 3 continued on next page

3c) Compute the spatial wavefunction, $\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp\, \phi(p) e^{-ipx/\hbar}$

d) How would you describe the properties of the particle corresponding to such a wave packet? (Eg, How well localized is the particle in space? What is its momentum?)

4. **a)** Consider the function

$$\psi(x) = \begin{cases} N & |x| \leq x_0 \\ 0 & |x| > x_0 \end{cases} .$$

Plot $\psi(x)$ and find the value of N that normalizes ψ (ie, so that $\int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = 1$).

b) $\psi(x)$ can be written as $\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp\, \phi(p) e^{ipx/\hbar}$. Find $\phi(p)$.

c) Show that $\int_{-\infty}^{\infty} dp\, |\phi(p)|^2 = 1$. (Hint: use $\int_{-\infty}^{\infty} dx\, \frac{\sin^2 x}{x^2} = \pi$.) We know that $|\psi(x)|^2$ is interpreted as the probability density for finding the particle at point x; how is $|\phi(p)|^2$ to be interpreted?

d) Since $|\psi(x)|^2$ is interpreted as the probability density for finding the particle at point x, if you were to measure the position x and compute x^n for many different particles, each with this wave function, the average value of the measurement would be $\langle x^n \rangle = \int_{-\infty}^{\infty} dx\, x^n |\psi(x)|^2$. Compute $\langle x \rangle$, $\langle x^2 \rangle$ and the root mean square position, $\Delta x \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

e) Use the formula $\langle p^n \rangle = \int_{-\infty}^{\infty} dp\, p^n |\phi(p)|^2$ to compute $\langle p \rangle$, $\langle p^2 \rangle$ and the root mean square momentum, $\Delta p \equiv \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.

f) What do you find for the product $\Delta x \Delta p$? Comment.