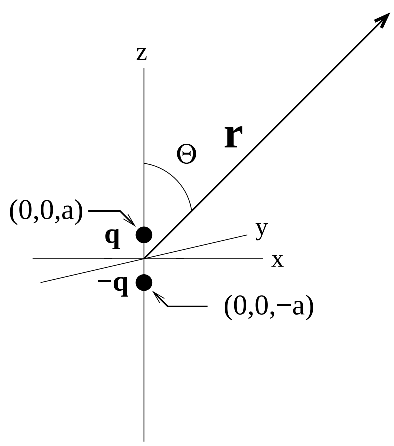
Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§12.1: 3, 16

Assigned problems. *All are to be turned in; the four problems from Boas will be graded, and will be worth 20 points each. Problem A below will be graded and is worth 20 points. Problem B is extra credit.*

§13.9: 9, 25 (the ends of the string of length ℓ are fixed)
§12.1: 6, 13

A. (20 points)



A pointlike charge q is located at position $a\hat{z}$; and a charge $-q$ is located at position $-a\hat{z}$. Thus the charge density is $\rho(\vec{r}) = q\,[\delta^3(\vec{r} - a\hat{z}) - \delta^3(\vec{r} + a\hat{z})]$.

(a) Write down the solution for the electric potential $\phi(\vec{r})$, given that it satisfies Poisson's equation, $\nabla^2\phi = k\rho$ where ρ is the charge density and k is a constant. Assume $\phi(\vec{r})$ vanishes for $|\vec{r}| \to \infty$. (Recall our class discussion of the potential due to a point charge at the origin).

Problem A continued $\Longrightarrow$

(b) Define $r \equiv |\vec{r}|$ and Taylor expand ϕ to leading nonzero order in the ratio a/r, which is small if r is very big (that is, if you are measuring ϕ far from the charges). Express your answer in terms of a, r and θ, where θ is the polar angle shown in the above picture.

B. (Extra credit)

The differential equation satisfied by the Hermite function $H_n(x)$ is

$$H_n'' - x^2 H_n = -(2n+1)H_n \ , \tag{1}$$

where n is an integer. Up to a rescaling of variables, this is the equation we solved in class for the quantum mechanical harmonic oscillator using operator methods (see also Chapter 12, §22 in Boas). Try solving this equation by using a series expansion

$$H_n(x) = e^{-x^2/2}(a_0 + a_1 x + a_2 x^2 + \ldots) \tag{2}$$

and finding a general recursion relation among the a's. You should find that the a's can all be zero above a certain point (for integer n only).

Do your answers agree with the operator results (eq. 22.13 in Boas) for H_0, H_1, H_2? If you hadn't been given the above suggestion for the expansion, how might you have come up with the prefactor of $e^{-x^2/2}$ in front of a polynomial, as the appropriate expansion?