Physics 228, Spring 2001

Problem set #6

Example problems. Not to be turned in - solutions given in the companion volume of solved problems:

Assigned problems. All are to be turned in; four problems from Boas will be graded, and will be worth 20 points each. Problem A below will be graded and is worth 20 points. Problem B is extra credit.

A. (20 points) Consider the earth to be a flat semi-infinite slab, with a surface temperature $u(x = 0, t) = u_0 + u_1 \cos \omega t$, where $\omega = 2\pi/(24 \text{ hours})$. This represents the daily surface temperature variation about a mean temperature u_0 . Given that the temperature in the earth obeys the heat equation

$$\nabla^2 u = \frac{1}{\alpha^2} \frac{\partial u}{\partial t} \; ,$$

find the temperature below the surface as a function of depth and time. In terms of α find how deep you have to go to first find that the temperature is exactly out of phase with the surface temperature (eg, that it is warmest in the middle of the night).

B. (Extra credit) Use Mathematica to plot your answer for the Boas problem Ch. 13, 4.2 assigned above, for h = 1, $\ell = 10$, v = 1 and for times t = 0 to $t = 2\ell/v$, preferably in steps of size $.02\ell/v$. (When I do this on a 700MHz PC, approximating the solution by the first 40 terms in the Fourier series, generating the 101 frames takes about 3 minutes.) If you create these figures in a Do loop, then select them and choose "Animate Selected Graphics" under the "Cell" Menu, you can watch the a movie of the wave in motion.

Please just turn in the frames for $t = 0, t = .3\ell/v, t = .5\ell/v, t = .7\ell/v$.