Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§8.7: 8, 16
§8.8: 7

Assigned problems. *All are to be turned in; three problems from Boas will be graded, and will be worth 20 points each. Problem A below will be graded and is worth 40 points. Problem B is extra credit.*

§8.7: 3, 5, 10, 15
§8.8: 31, 34

A. (40 points) Three objects of mass m lie on a frictionless track and are connected to each other in a line by two identical springs of spring constant k (object 1 is connected to object 2 by one spring, object 2 is connected to object 3 by another spring). When not stretched or compressed, the springs have length L. The motion of these objects is one dimensional as they are restricted to lying on the track. The positions of the three masses (from left to right) are x_1, x_2, and x_3 respectively.

a. Write down the potential energy U in terms of the three variables Δ_i, where $x_1 = \Delta_1 - L$, $x_2 = \Delta_2$, $x_3 = \Delta_3 + L$.

b. The equations of motion for the three variables are given by

$$m\ddot{\Delta}_i = -\frac{dU}{d\Delta_i} \ .$$

Write these three coupled equations in matrix form.

c. By diagonalizing the equation of motion, find the three normal modes for this problem, and their solutions. Describe the motion of the masses for each of these normal mode solutions.

d. Suppose that at $t = 0$ $\dot{x}_1 = \dot{x}_3 = 0$, $x_1 = -L$, $x_3 = +L$, $x_2 = 0$ and I kick object 2 such that $\dot{x}_2 = v_0$. Find the solution for subsequent motion of the objects.

B. (Extra credit) Find the general solution to the following pair of coupled *nonlinear* first order differential equations for $n_1(t)$ and $n_2(t)$.:

$$\dot{n}_1 = \lambda_1(1 - n_2/N_0)n_1 \ , \qquad \dot{n}_2 = \lambda_1(n_1/N_0 - 1)n_2 \ , \tag{1}$$

where N_0 is a constant. Hint: note that if the nonlinear terms vanished, then the solutions would be $n_1 = c_1 e^{\lambda_1 t}$, $n_2 = c_2 e^{-\lambda_1 t}$, so try substituting into the above equations the change of variables

$$n_1(t) = N_0 e^{\lambda_1 t} f_1(t) \ , \qquad n_2(t) = N_0 e^{-\lambda_1 t} f_2(t) \tag{2}$$

and then find a way to solve for f_1 and f_2. What do the solutions look like with the initial conditions $n_1(0) = 2N_0$, $n_2(0) = N_0/2$?