Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§8.5: 2, 24
§8.6: 13, 25, 41

Assigned problems. *All are to be turned in; three problems from Boas will be graded, and will be worth 20 points each. Problem A below will be graded and is worth 40 points; three additional problems from Boas will be graded at 20 points each.*

§8.5: 4, 26, 31(c,d), 40
§8.6: 11, 24, 39

A. (40 points) In this problem, you will consider the linear differential equation

$$\dot{y}(t) + ay(t) = f(t)$$

where a is a positive constant.

a. Find the general solution to the above differential equation.

b. Find the solution for $y(t)$ for the case $f(t) = e^{i\omega t}$.

c. Use Fourier analysis to find the Green function $G(t, t_0)$ for this equation, where G satisfies

$$\frac{d}{dt}G(t, t_0) + aG(t, t_0) = \delta(t - t_0)$$

and $G(t, t_0) = 0$ for $t < t_0$. Hint: use the result of part (b), along with the fact

$$\delta(t - t_0) = \int_{-\infty}^{\infty} e^{i\omega(t-t_0)} \frac{d\omega}{2\pi} .$$

You will need the integral

$$\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{\omega - ia} \frac{d\omega}{2\pi} = \begin{cases} 0 & t < 0 \\ ie^{-at} & t > 0 \end{cases}$$

d. Use your general solution from part (a) with $f(t) = \delta(t - t_0)$ to check whether you got the correct solution for G in part (c).

e. Show explicitly that the general solution to the differential equation for arbitrary $f(t)$ can be written as

$$y(t) = y_h(t) + \int_{-\infty}^{\infty} G(t, t_0) f(t_0) dt_0$$

Where y_h is the general solution to the homogeneous equation

$$\dot{y} + ay = 0 \ .$$

That is, show that this reproduces your answer for part (a).

The above first order differential equation is simple enough that the technique of Green functions is overkill; the technique is more useful for solving second order partial differential equations. However, I gave it as an example so that you could easily check that the Green function technique gives the correct answer. In many problems, the calculation you did for part (c) is the easiest way to solve the inhomogeneous equation. The method is used in practically all branches of physics.