Using *Mathematica* to plot solutions to Laplace's equation

The problem: solve Laplace's equation

$$\nabla^2 \phi = 0$$

in two dimensions for a rectangle of size a in the x direction, size b in the y direction, with boundary conditions:

$$\phi(x,0)=\phi(0,y)=\phi(a,y)=0, \qquad \phi(x, b) = v.$$

The solution: $\phi = \sum_{n=1}^{\infty} c_n \text{Sin}[n \pi x/a] \text{Sinh}[n \pi y/a]$, **where**

$$c_n = \phi_0 \left(1 - (-1)^n\right) \frac{2}{n \pi \text{Sinh}[n \pi b/a]}$$

■ **First I define the approximate solution as the sum of the first 40 terms:**

In[16]:=

```
nmax = 40;
c[n_] = v (1 - (-1)^n) 2 / (n π Sinh[n π b / a]);
ϕ = Sum[c[n] Sin[n π x / a] Sinh[n π y / a], {n, 1, nmax}];
```

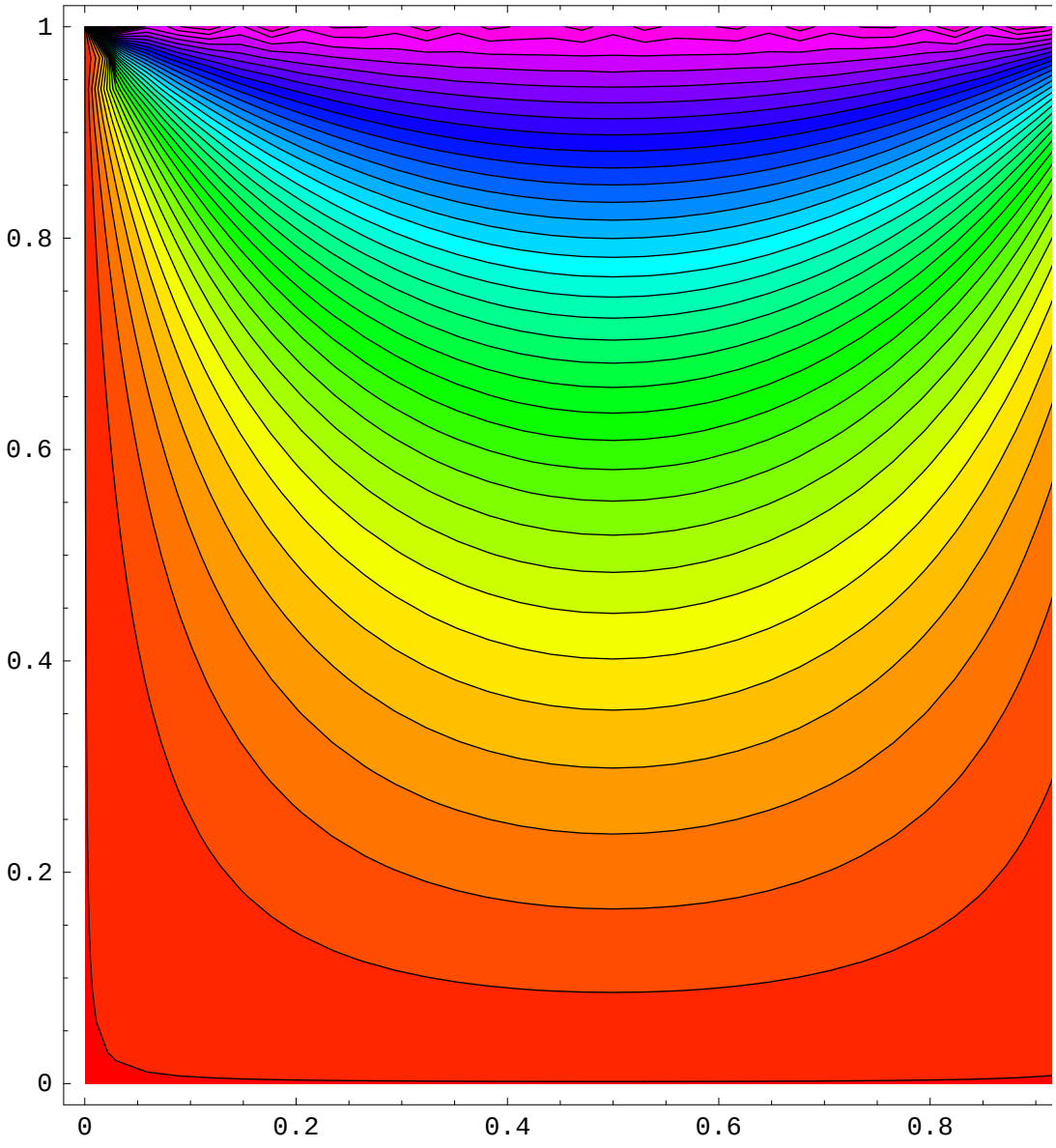
■ **Next I choose for definiteness $a = b = v = 1$**

In[19]:=

```
v = 1;
a = 1;
b = 1;
```

■ **Next I have the countour plot made. Look under Help for CountourPlot, and it will tell you about the options this function can take.**

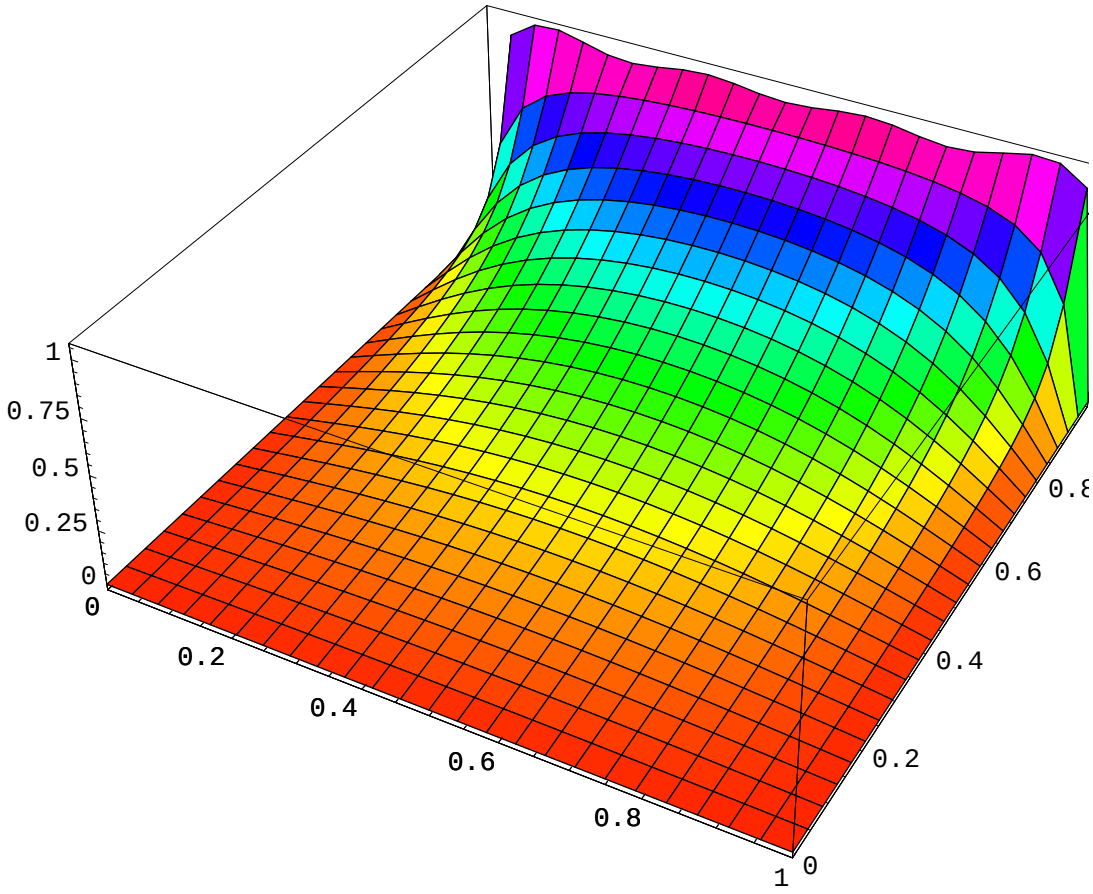
In[22]:=
```
ContourPlot[ϕ, {x, 0, a}, {y, 0, b},
 PlotPoints → 35, Contours → 40, ColorFunction → Hue]
```



Out[22]=
```
- ContourGraphics -
```

■ **Next I plot the 3D graph of the function:**

In[23]:=
```
Plot3D[ϕ, {x, 0, a}, {y, 0, b}, ColorFunction → Hue, PlotPoints → 25]
```



Out[23]=
```
- SurfaceGraphics -
```