Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§7.5: 2, 7
§7.7: 2, 7
§7.8: 1, 11
§7.9: 11
§7.11: 7

Assigned problems. *To be turned in. Problem A below will be graded; four additional problems chosen at random from those in Boas will be graded, worth 20 points apiece. Problem B is extra credit.*

§7.5: 5, 9
§7.7: 9
§7.8: 5, 12b (12b: complex exponential series only)
§7.9: 9
§7.11: 6

A. (20 points) Compute the following integrals, where m and n are integers. For parts (b)-(d), first write

$$\cos mx = \frac{1}{2}\left(e^{imx} + e^{-imx}\right) , \qquad \sin mx = \frac{1}{2i}\left(e^{imx} - e^{-imx}\right) ,$$

and similarly with m replaced by n.

a. $\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-imx}e^{inx}\,\mathrm{d}x$

b. $\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos mx \cos nx\,\mathrm{d}x$

c. $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin mx \sin nx\,\mathrm{d}x$

d. $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin mx \cos nx\,\mathrm{d}x$

B. (Extra Credit – highly recommended)

Consider the function $f(x) = x$ on the interval $-\pi < x \leq \pi$, repeated over the real line with period 2π. This is an odd function, so it will have a Fourier sine series expansion. Use Mathematica to find the Fourier coefficients for this expansion. Plot the sum of the first 2, 5, 10, and 50 terms in this series on the interval $-2\pi \leq x \leq 2\pi$.

I recommend doing this problem because it really let's you see how the Fourier expansion converges. Notice that with the first 50 terms, you can reproduce the function very well, except for near the sharp corners. Why do you think that is?