Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§3.4: 18, 21, 24

Assigned problems. *To be turned in. Problems A and B below will be graded; two additional problems chosen at random from those in Boas will be graded, worth 20 points apiece.*

§3.4: 12, 17, 19

A. (40 points) Consider the real, symmetric (and therefore Hermitean) matrix

$$M = \begin{pmatrix} 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{2} & \frac{1}{2\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} \end{pmatrix} \tag{1}$$

a. Find the eigenvalues of M.

b. Find the normalized eigenvectors of M. Show that the three eigenvectors are mutually orthogonal.

c. Find a matrix P such that $P^{-1}MP = D$, where D is diagonal.

d. Compute the matrix M^{101}. Also compute $\cos \pi M/2$ and $\sin \pi M/2$.

B. (20 points)

Under a Lorentz boost in the x direction, the time and space coordinates $\{t, x\}$ become transformed to $\{t', x'\}$. If we write the transformation in matrix language,

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \Lambda \begin{pmatrix} t \\ x \end{pmatrix} , \tag{2}$$

where Λ is a real, 2×2 matrix. Lorentz boosts preserve "interval":

$$t^2 - x^2 = t'^2 - x'^2 \ . \tag{3}$$

[Note that this looks a little like rotations in the $x - y$ plane which preserve distance: $(x^2 + y^2) = (x'^2 + y'^2)$ – except now there is a minus sign.] Given the above constraint, what is the most general form the matrix Λ can take? Show that your solution satisfies

$$\Lambda^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ . \tag{4}$$