Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

[§3.4: 18, 21, 24] $\Longleftarrow$ **Postponed until problem set 7**
§10.4: 13, 22, 36

Assigned problems. *To be turned in. Problems A, B, and C below will be graded; two additional problems chosen at random from those in Boas will be graded. All graded problems are worth 20 points each. Problem D is extra credit: doing extra credit problems do not change your score, but at the end of the term can make the difference in borderline grade assignments.*

[§3.4: 12, 17, 19] $\Longleftarrow$ **Postponed until problem set 7**
§10.4: 14, 19, 34, 38

A. (20 points) Consider the vector $\vec{v} = (3, -4)$ in the $x - y$ plane. Let $\vec{w}$ be related to $\vec{v}$ by a rotation in the $x - y$ plane:

$$\vec{w} = R(\theta)\vec{v} \ , \qquad R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \ . \tag{1}$$

What is the angle θ for which $\vec{w}$ is pointing along the positive x-axis?

B. (20 points) Consider the matrix

$$M = \begin{pmatrix} a & b & 0 \\ 0 & (a+b) & 0 \\ b & -b & (a-b) \end{pmatrix} \ . \tag{2}$$

where a and b are variables.

(a.) What is the condition on a and b that must be fulfilled if M^{-1} is to exist?

(b.) Compute the eigenvalues of M.

(c.) Compute the normalized eigenvectors of M.

(d.) Compute a similarity transformation matrix P, such that $P^{-1}MP$ is diagonal.

C. (20 points)

(a.) Prove that if A, B, are matrices of dimensions such that the product (AB) is defined, then $(AB)^\dagger = (B^\dagger A^\dagger)$. Remember that for a matrix M, $M^\dagger \equiv (M^*)^T$. Explain why how this generalizes for more matrices: $(ABC\cdots)^\dagger = ?$

(b) For the rest of this problem, you will consider the properties of a Hermitean matrix H: $H = H^\dagger$. Suppose the vector v is an eigenvector of H with eigenvalue λ, so that $Hv = \lambda v$. Assuming λ to be complex, what is $v^\dagger H$ equal to? (Hint: use the result of part (a)).

(c) For the same Hermitean matrix H and eigenvector v as in (b), prove that all eigenvalues λ are real by considering the quantity $v^\dagger H v = (v^\dagger H)v = v^\dagger(Hv)$.

(d) Suppose that v_1 is an eigenvector of the hermition matrix H corresponding to the eigenvalue λ_1, and v_2 is another eigenvector of H corresponding to the eigenvalue λ_2. Prove that if $\lambda_1 \neq \lambda_2$, then $v_1^\dagger v_2 = v_2^\dagger v_1 = 0$. That means that these complex eigenvectors v_1 and v_2 are orthogonal.

D. (Extra credit) Use Mathematica to compute the trace, determinant, inverse, eigenvectors and eigenvalues of the matrix

$$M = \begin{pmatrix} 2 & 0 & 2i & 0 & 1 \\ 0 & -1 & 0 & -2i & 0 \\ -2i & 0 & 1 & 1 & 1 \\ 0 & 2i & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 & -1 \end{pmatrix} \tag{3}$$