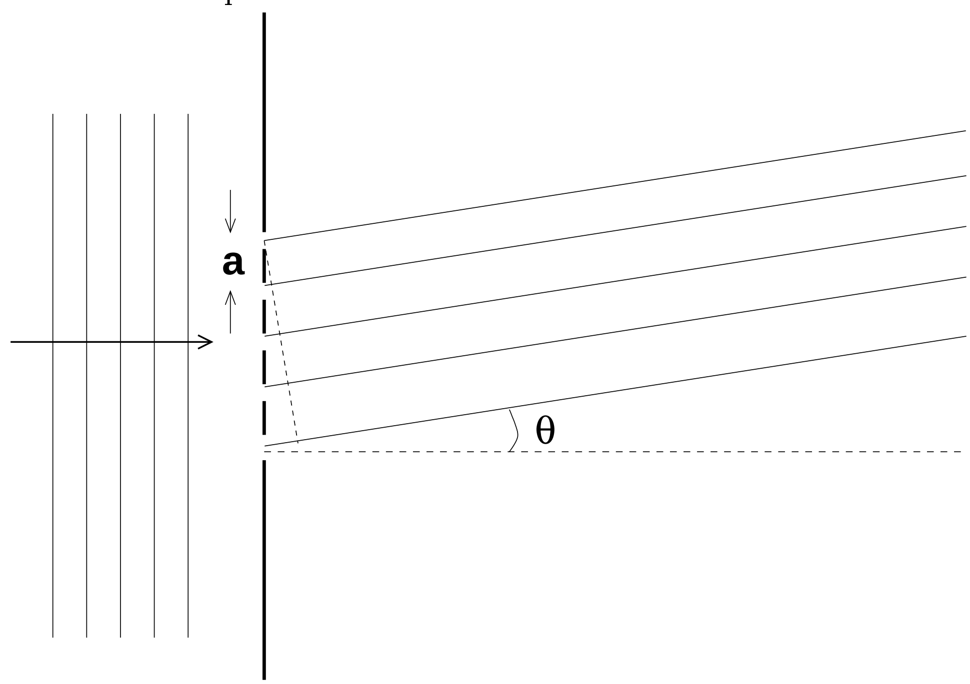
Physics 227, Winter 2001 **Problem set #4** Due: Wed. 1/31/01

One assigned problems. *To be turned in and graded.*

A. (100 points)
Consider light with wavelength λ passing through a grating with 5 narrow slits a distance a apart.



The amplitude of the light on a distance sceen, at an angle θ from the perpendicular to the grating, is the sum of the contributions from each of the slits:

$$E = E_0 \sin \omega t + E_0 \sin(\omega t - \delta) + E_0 \sin(\omega t - 2\delta) + E_0 \sin(\omega t - 3\delta) + E_0 \sin(\omega t - 4\delta) \quad (1)$$

where

$$\delta = 2\pi \frac{a \sin \theta}{\lambda} \quad (2)$$

The phase δ accounts for the different number of wavelengths that fit along the paths taken by a light rays between the screen at angle θ, and the slits through which they passed.

a. Show that

$$E(\theta, t) = E_0 \,\mathrm{Im}\left[\sum_{n=0}^{4} e^{i(\omega t - n\delta)}\right] \tag{3}$$

and compute it as a compact, real expression. Hint: recall the formula for finite geometric series in Chapter 1.

b. Compute the (brightness) $I(t)$ of the light on screen given by

$$I(\theta, t) = |E(\theta, t)|^2 \tag{4}$$

c. This intensity is actually oscillating very rapidly over time. What our eye sees is the average intensity over one oscillation period, $T = 2\pi/\omega$. Compute the time averaged intensity,

$$\langle I(\theta)\rangle = \frac{1}{T}\int_0^T I(\theta, t)\, dt \tag{5}$$

d. Carefully plot the time averaged intensity $\langle I(\theta)\rangle$ on the screen, as a function of angle θ. How many bright spots (local maxima in $\langle I(\theta)\rangle$) appear on the screen? Where are the dark spots? Physically, what is causing the dark spots?