Physics 227, Winter 2001 **Problem set #3** Due: Wed. 1/24/01

Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§2.5:	46
§2.7:	16
§2.9:	24
§2.10:	16, 22
§2.11:	5

Assigned problems. *To be turned in. Problems A and B below will be graded; three additional problems chosen at random from those in Boas will be graded. All graded problems are worth 20 points each. Problem C is extra credit: doing extra credit problems do not change your score, but at the end of the term can make the difference in borderline grade assignments.*

§2.5:	47
§2.7:	15
§2.9:	18
§2.10:	11, 21
§2.11:	9
§2.12:	8
§2.14:	22

A. (20 points)

Suppose $z(t) = x(t) + i\,y(t)$ where $x(t)$ and $y(t)$ are real functions of t, and that $z(t)$ satisfies the differential equation

$$\ddot{z} + 2b\dot{z} + \omega^2 z = Fe^{i\omega_0 t} \ , \qquad (1)$$

where b, ω and ω_0 are real parameters. What is the differential equation satisfied by $x(t)$? *You aren't being asked to solve this differential equation, which corresponds to a damped, driven harmonic oscillator.*

B. (20 points)

In an AC electrical circuit one might have an impedance $Z = R - \frac{i}{\omega C}$ and a driving voltage $V(t) = V_0 e^{i\omega t}$, where R, C, ω, and V_0 are all real parameters. The current $I(t)$ satisfies

$$I(t) = V(t)/Z \ . \tag{2}$$

Write $I(t)$ as

$$I(t) \equiv I_0 e^{i\theta(t)} \ , \tag{3}$$

where I_0 and $\theta(t)$ are real. Find I_0 and $\theta(t)$ as functions of R, C, ω, V_0, and t.

C. (Extra credit)

Consider the function $f(\omega) = \frac{1}{\omega - \omega_0 - i\gamma}$ where ω_0 and γ are real parameters and ω is a real variable.

a. Find the real and imaginary parts of $f(\omega)$.

b. Use Mathematica to plot from $\omega = 0$ to $\omega = 10$ the real and imaginary parts of $f(\omega)$ for $\omega_0 = 5$, $\gamma = 1$, using the `Plot`, `Re` and `Im` functions. Remember that in Mathematica the $\sqrt{-1}$ is written as `I`, not `i`.

Note: Problem sets must be turned in by the end of class, or must be in David Kaplan's Physics Department mailbox by 12:20 PM on the day they are due. Solutions will be posted on the web.