Physics 227, Winter 2001 **Problem set #2** Due: Wed. 1/17/01

Example problems. *Not to be turned in — solutions given in the companion volume of solved problems:*

§1.13:	11
§1.15:	17
§2.4:	11, 17
§2.5:	6, 43

Assigned problems. *To be turned in. Problems A and B below will be graded; three additional problems chosen at random from those in Boas will be graded. All graded problems are worth 20 points each. Problem C is extra credit: doing extra credit problems do not change your score, but at the end of the term can make the difference in borderline grade assignments.*

§1.13:	15
§1.15:	21, 28
§2.4:	3
§2.5:	33

A. (20 points)

Remember that a particle of mass M on a spring with spring constant k and equilibrium length x_0 obeys Newton's law

$$M\ddot{x} = -\frac{\mathrm{d}U}{\mathrm{d}x}$$

where $U(x)$ is the potential energy

$$U(x) = U_0 + \frac{1}{2}k(x - x_0)^2 \ ,$$

where the value of the constant U_0 is irrelevant to the particle's motion. You have seen that the resultant motion is a steady oscillation with frequency $\omega = \sqrt{k/M}$.

Suppose instead that we consider the motion of a particle of mass M in a potential

$$U(x) = -\frac{a}{x} + \frac{b}{x^2}$$

where a and b are positive dimensionful constants.

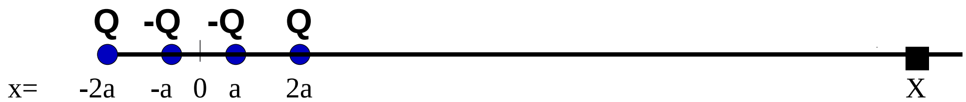
a. Sketch the potential $U(x)$.

b. Find the "equilibrium position" x_0 in terms of a and b. This is the position where the particle feels no force.

c. Perform a Taylor expansion of $U(x)$ about $x = x_0$ to order $(x - x_0)^2$. Compare your approximate expression for U with the example of a particle on a spring, and thereby determine the oscillation frequency of the particle if it makes small oscillationsin the potential $U(x)$ about the equilibrium position x_0.

B. (20 points)

Suppose four charges are placed on the x-axis: charges $+Q$ are located at positions $x = +2a$ and $x = -2a$, while charges $-Q$ are located at positions $x = +a$ and $x = -a$.



a. Show that the electric potential $V(X)$ at a point $x = X > 2a$ on the x-axis is given by

$$V(X) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{X-2a} + \frac{1}{X+2a} - \frac{1}{X-a} - \frac{1}{X+a}\right]$$

b. How fast does this potential fall off with X for large X (Like X^{-1}? X^{-2}? etc.) ? *Hint: a/X is a small dimensionless parameter you can expand in.*

C. (Extra credit) We have seen that the Taylor expansion for

$$1/(1+x) = 1 - x + x^2 - x^3 \ldots$$

with an interval of convergence $-1 < x < 1$.. Use Mathematica to plot on the same graph over the interval $0 \leq x \leq 2$ the functions

$$f(x) = \frac{1}{1+x}\,, \qquad f_2(x) = 1-x+x^2\,, \qquad f_6(x) = 1-x+x^2-x^3+x^4-x^5+x^6\,.$$

Comment on what you see.

Note: Problem sets must be turned in by the end of class, or must be in David Kaplan's Physics Department mailbox by 12:20 PM on the day they are due. Solutions will be posted on the web.