

SPIN EXCITATIONS IN NUCLEI

G.F. BERTSCH

Cyclotron Laboratory and Dept. of Physics
Michigan State University, East Lansing, MI 48824

Abstract: The recent progress in the study of nuclear spin excitations is reviewed. The (e,e') measurement of the giant M1 state in ${}^4_8\text{Ca}$ and the systematics of the giant Gamow-Teller state seen in the intermediate energy (p,n) reaction give a clear picture of the spin-dependent dynamics. The effective interaction strength is consistent with previous knowledge of the nuclear force. However, the excitation strengths are much smaller than shell-model theory predicts. This may be understood at least partially by considering additional hadronic degrees of freedom.

Introduction

There has been great progress in the last few years in the study of spin excitations, and at this conference there are several significant contributions. For a long time, there was only one nucleus whose spin properties we really knew, namely ${}^{12}\text{C}$. Now for the first time we have measurements of the spin excitation strength in magic nuclei with spin-unsaturated j-shell closures. The best example is ${}^4_8\text{Ca}$, where there are recent measurements both of the M1 strength by electron scattering¹⁾, and of the strength of the Gamow-Teller operator $\sigma\tau$ by charge exchange reactions²⁾. With such excellent quality data we can with confidence infer the properties of the residual interaction. We can also see a pattern emerging, with the spin operator becoming strongly quenched in nuclei.

Before I go into any detail on the interpretation of the experiments, I would like to remind you qualitatively what is learned. Measurement of the M1 strength tells us the degree to which spins are unpaired in the ground state. The excitation energy tells us the effective spin-spin interaction, which then should be usable to predict many other features of the spin degrees of freedom, such as higher multipoles, and interactions with spin-dependent probes such as pions.

M1 strength

According to the shell model, the M1 strength should be concentrated in a single particle-hole state for a closed shell nucleus with spin-unsaturated j-shell closures. The measurement on ${}^4_8\text{Ca}$ is particularly welcome for this reason. The experimental result is that the strength is indeed concentrated, in a state at 10.23 MeV excitation. This is seen in the data of Fig. 1:

The excitation energy of the ${}^4_8\text{Ca}$ M1 state is close to what is expected theoretically. I want to show you this in some detail, because there has been some controversy about ${}^{208}\text{Pb}$, as to where in energy the state should be. In the simplest theory, the excitation energy is due to the single particle energy of the $(f_{5/2}^{-1}f_{7/2}^{-1})$ configuration, together with the residual interaction. The single-

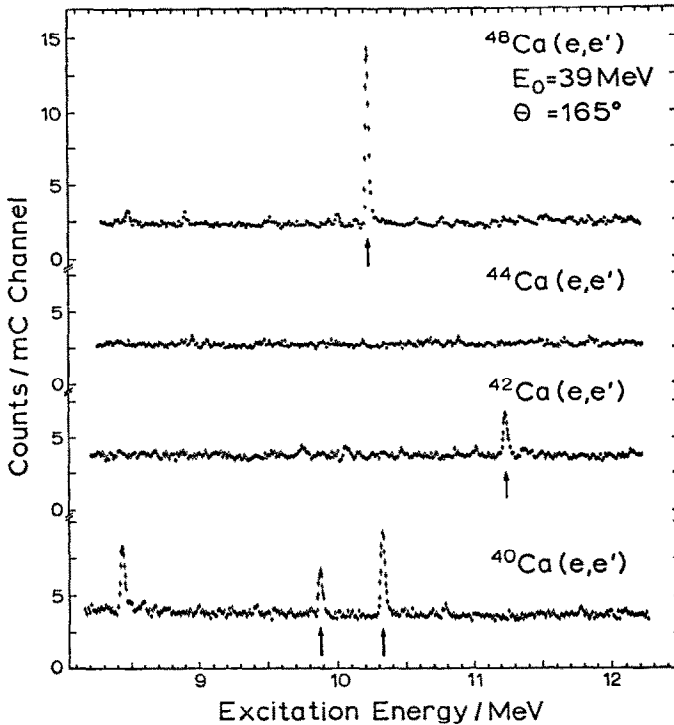


Fig. 1 Excitation spectrum for Ca isotopes in 165° electron scattering, measured by Steffen, et al.¹⁾

particle energies I determine empirically by binding energy differences:

$$e_{5/2^-} - e_{7/2^-} = 2BE(^8\text{Ca}) - BE(^9\text{Ca}, 5/2^-) - BE(^7\text{Ca}, 7/2^-) \quad (1)$$

From the nuclear mass tables and the experimental excitation of the $5/2^-$ state in ^9Ca , I then find a single-particle excitation energy of 8.4 MeV. This is larger than the presumed splitting of the spin-orbit potential. One reason is that the energy includes the loss of pairing energy in the ^8Ca ground state.

For the residual interaction, I will apply the Brueckner G-matrix interaction calculated with the Reid potential²⁾. The result for $(f_{5/2^-} f_{7/2^-}^{-1})$ is 1.9 MeV repulsive. Thus the state is predicted at 10.3 MeV, in agreement with experiment. We can make the same kind of analysis for other nuclei, with results shown in Table I. For ^{12}C , the method overpredicts the excitation energy. But ^{12}C is far from a closed $p_{3/2}$ shell nucleus, so we should not expect to do very well here. The nuclei ^{90}Zr and ^{208}Pb have been problem cases because strong concentrations of strength were never found at

Table I. Energetics of M1 states

Nucleus	Empirical Particle-Hole Energy	Residual Interaction ^{a)}	Total Excitation	Experimental Energy
^{12}C	13.8	3.4	17.2	15.1
^{48}Ca	8.8	1.9	10.6	10.23 ^{b)}
^{90}Zr	6.7	1.6	8.3	8 ^{c)}
^{208}Pb	5.6	1.8	7.4	7.5

a) Bare G-matrix from Reid potential (ref. 3).

b) Ref. 1.

c) Ref. 10.

the theoretical energies shown here.

However, the data on ^{48}Ca give us a new insight on the systematics of strength to expect. Let me first review with you the experimental situation for ^{208}Pb . There is a group of states at ≈ 7.5 MeV excitation, which together have 1/5 of the shell model limit of strength. There is also a state at 7.99 MeV, identified at one time as M1, which would bring the strength up to 40% of the shell model. The M1 identification was made by observing a polarization asymmetry in the photonuclear reaction⁴⁾. The electron scattering is no help here because a strong M2 state would mask the M1⁵⁾. But later higher resolution experiments showed that this state was E1^{6,7)}. We can see the proof easily in (γ, n) data shown in Fig. 2.

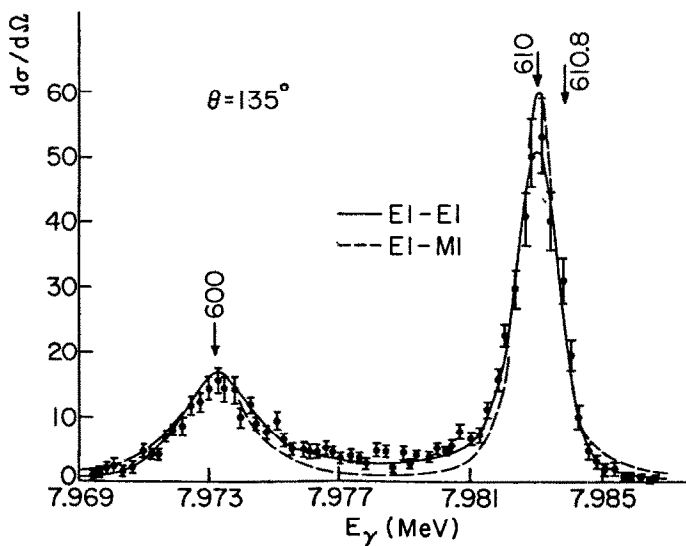


Fig. 2 Cross section for the $^{208}\text{Pb}(\gamma, n_0)$ reaction in the vicinity of the 7.99 MeV state, from ref. 7.

The lower peak is known to be E1, and the upper peak is the state in question. If it is M1, there would be no interference between the two states. But examining the cross section halfway between the states, we see a plateau that could only be due to constructive interference. Thus the state is E1, and there is only 20% of the shell-model strength in the expected energy region. The quenching phenomenon has been known for some time from the systematics of low-lying transitions, particularly the suppression of M1 transitions around $^{208}\text{Pb}^{36}$). Moreover, the quenching appears to be quite general, with the spin operator suppressed in the M2 strength as well^{5,14}).

The new data on ^4Ca show a strength of 1/3 of the shell model limit. In ^{12}C the reduction is similar, but for ^{12}C the fault is with an oversimplified shell model based on a closed $p_{3/2}$ shell. In fact, the ^{12}C strength agrees quite well with a more complete shell model calculation¹¹). So one possibility for this quenching is that even heavy nuclei have strong ground state correlations. These correlations are treated in the RPA theory and their effect is calculated to be small, reducing the strength only by a factor $(\epsilon_p - \epsilon_n)/E$. Another possibility, suggested by Brown, et al.¹²), is that the single-particle energies should be somewhat larger, moving the center of gravity of the strength to a higher energy. This does not provide a good explanation for the large quenching. For one thing, the spin-orbit potential is known independently from elastic proton scattering, and is consistent to within one MeV of the empirical bound single-particle energy splittings. In heavy nuclei, there are theoretical grounds for thinking that the spin-orbit splitting should even decrease. Also, the correlations that would shift the single-particle strength to nearby energies would not tend to shift the spin-flip strength if the correlations were isoscalar. We shall see evidence later that this is so.

One further possibility to explain the quenching is that the other degrees of freedom that we omit in describing the nucleus as a collection of neutrons and protons, are important for the spin operator. I will return to this later.

Gamow-Teller Strengths

We have known for some time from charge exchange reactions that the strength of the σ operator is concentrated in a peak above the analog state¹⁵). But with the recent use of the (p,n) reaction above 100 MeV, we have for the first time a really clean tool to study this strength. The reason may be seen in Fig. 3, showing the neutron-proton differential cross section¹⁶). There is a peak at 180° , corresponding to forward charge exchange, of magnitude 11 mb/sr. An amplitude analysis of potential models shows that most of the cross section is spin-flip. Thus the operator σ is enhanced for low momentum transfers, which is perfect for a probe to study the Gamow-Teller strength.

As an example of the kind of data obtained, we see in Fig. 4 the reaction of ^4Ca , from Anderson, et al.²). The major peaks at 0° are: the 0^+ analog of the ^4Ca ground state, the analog of the M1 seen in electron scattering, a low 1^+ and the broad 1^+ of the Gamow-Teller giant state. The identification is clear because all of these peaks fall off sharply with angle away from 0° . Also at this conference are reported measurements of many other heavy nuclei. For my analysis I will emphasize the magic nuclei, ^{90}Zr (ref. 10) and ^{208}Pb (ref. 17). The spectrum for ^{90}Zr is shown in Fig. 5. In both cases the Gamow-Teller state is a prominent peak at

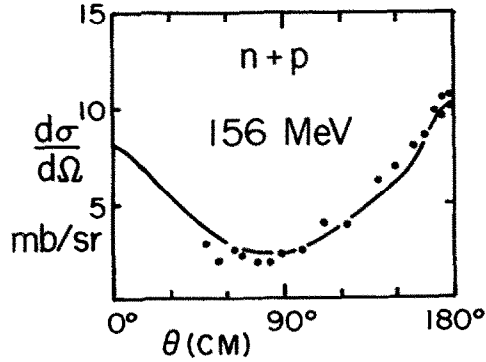


Fig. 3 The neutron proton differential cross section, from ref. 16.

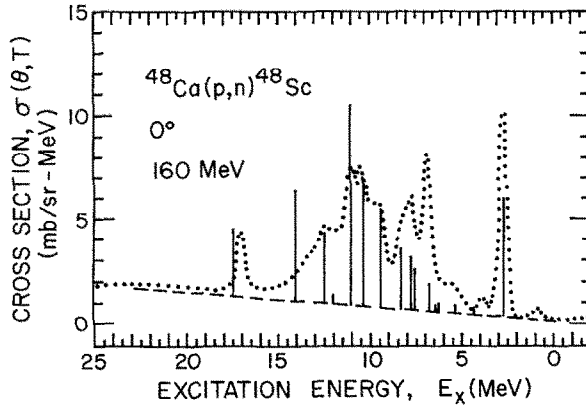


Fig. 4 The reaction $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ at 0° , measured by Anderson, et al. (ref. 2). The peaks at 2.52 MeV, 6.67 MeV and 16.81 MeV correspond to the low 1^+ state, the ground state analog and the M1 analog state, respectively. The broad peak centered at 10.4 MeV is the Gamow-Teller giant strength. A theoretical strength function¹⁹⁾ is shown by the vertical lines.

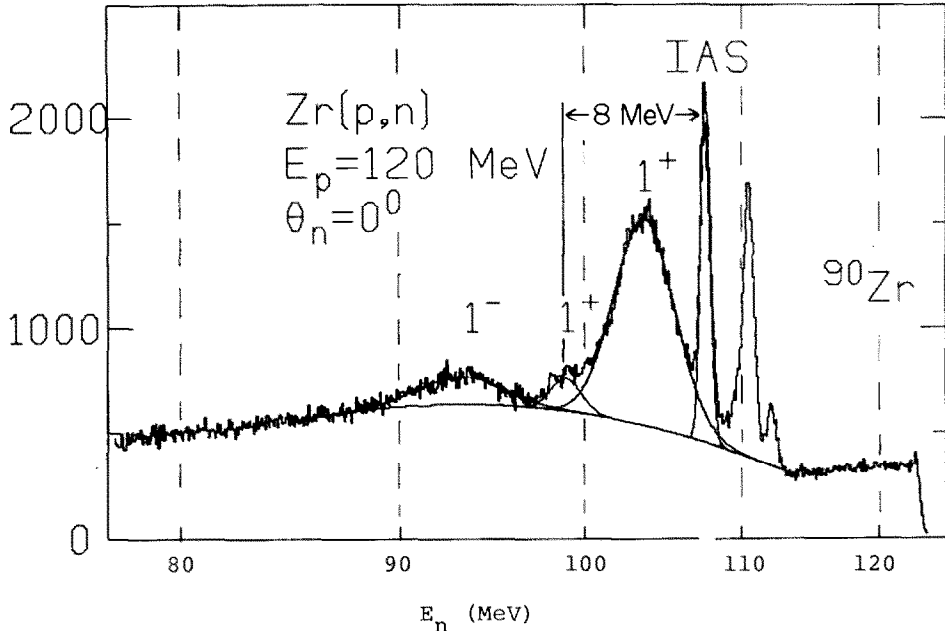


Fig. 5 The spectrum of neutrons in the reaction $^{90}\text{Zr}(p,n)$ measured by Sterrenburg, et al. (ref. 10).

0° , located somewhat above the target analog. The width of the peak is about 4 MeV.

Theoretically, the Gamow-Teller strength was long predicted to concentrate in a high energy peak¹⁸⁾. In Figure 6 are depicted the various stages of theoretical treatment. On the crudest level, given by the independent particle model, there are two pieces to the strength function, $(f_{5/2}^{-1}f_{7/2})$ and $(f_{7/2}^{-1}f_{7/2})$, carrying roughly equal strength. The excitation energy of the upper state is due to the spin-orbit potential. On the next level of approximation, the residual interaction is included in the Hamiltonian. Both states are pushed up, and the bulk of the strength goes to the upper state. Finally, in the most detailed theoretical treatment the mixing of the particle-hole states with more complicated configurations can be included. In the calculation of Gaarde, et al.¹⁹⁾, this last step fractionates the upper peak into many pieces, but leaves the lower peak unaffected. And this seems to be just what the experiment shows in ^{40}Ca .

I will go back now to the second stage, the inclusion of the residual interaction, because it is here that we learn about the spin-dependent force. But first, it is important to know the spin-orbit splitting if the residual interaction is to be accurately tested. I calculate the spin-orbit splittings with a Woods-Saxon potential of the usual type. I take the well geometry and spin-orbit strength from the global fit to low-energy proton scattering data by Becchetti and Greenlees⁸⁾. Roughly the same results are obtained using a common geometry and a spin orbit strength of 15 MeV fm^2 . I first compare the excitation energy of the Gamow-Teller

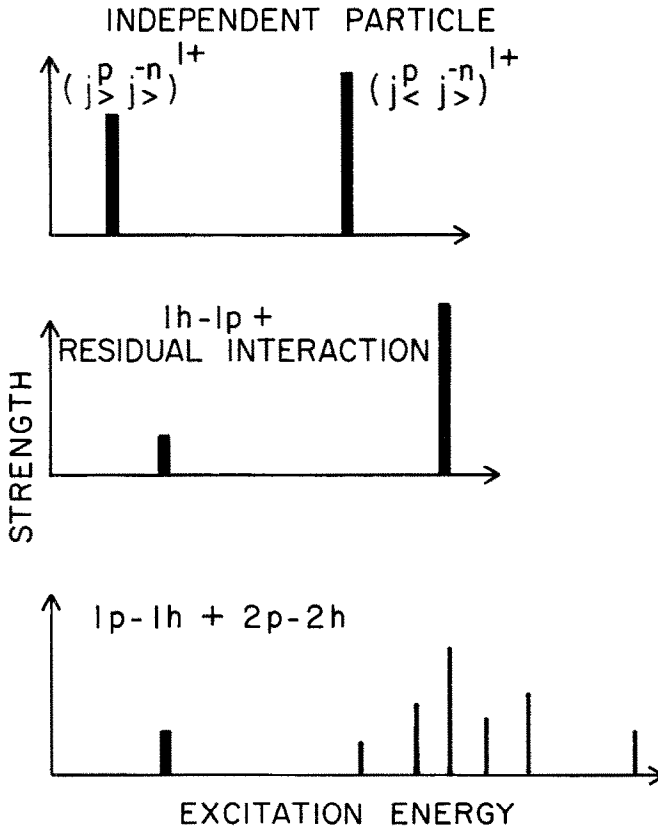


Fig. 6 Schematic Gamow-Teller Strength Function for heavy nuclei. The first graph shows the independent particle-model, for which the spin orbit splitting alone provides the high energy component. The second graph shows the effect of the residual interaction among the particle hole states. The lower graph shows the effect of mixing in more complicated configurations as in ref. 5.

state with the spin-orbit parameter excitation. The difference is the measure of the residual interaction. These numbers are shown in Table II, for the three magic nuclei that have been measured^{2,13,17}). As with the M1 states, this can be compared with the residual interaction calculated from the Brueckner theory. From the next-to-bottom row of the table, we see that these numbers are close to the empirical data.

Table II. Energetics of Giant Gamow-Teller States

	Nucleus		
	⁴⁸ Ca	⁹⁰ Zr	Pb
Experimental Excitation Energy (jj^{-1}) particle- hole excitation	10.4 MeV ^{a)}	9.3 ^{b)}	15.6 ^{c)}
Spin-Orbit Splitting	6.1 MeV	6.4	6.3
	<u>Residual Interaction</u>		
Empirical	4.5 MeV	3.0	5.7
G-Matrix	3.5 MeV	3.0	
G'_0	1.3	1.2	1.6

a) Ref. 2.

b) Ref. 10, 37.

c) Ref. 17.

I now want to turn the analysis around and use the systematics of the excitation energy to deduce the residual interaction. It is then necessary to make a simplified parameterization of the interaction. Using the parameterization scheme of Bohr and Mottelson, C. Gaarde²³⁾ has found that the energy systematics can be well accounted for. I prefer to use the Landau-Migdal scheme, since this is easier to connect with other physical phenomena. The idea is to parameterize the interaction with a zero-range form that reproduces the important properties of the true effective interaction. The advantage is that calculations are easy; in fact I did the calculations shown here in an afternoon with a table of Woods-Saxon wavefunctions and a hand calculator. The disadvantage is that each type of excitation requires its own Landau parameter. The relation between the usual Landau parameter G'_0 and a delta function interaction is

$$v_{\text{eff}} = G'_0 \frac{\hbar^2 \pi^2}{2mk_F} \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \delta(r_1 - r_2) \quad (2)$$

with

$$\frac{\hbar^2}{2m} \frac{\pi^2}{k_F} \approx 150 \text{ MeV-fm}^3.$$

This type of calculation has been done for spin excitations in the Pb region by Speth and collaborators^{21,22}). If we use the parameter G'_0 to fit the energies of the Gamow-Teller peaks, we find the numbers given in the last row of Table II. These measurements of G'_0 are not as consistent with each other as could be desired. However, it is clear that the parameter is larger than one and repulsive. The Brueckner interaction, since it gives about the right energy shifts, should have associated a G'_0 in the right range. In fact, the calculation of G'_0 directly from the Brueckner theory in infinite nuclear matter yields a similar number²³).

The measurement of G'_0 has significance beyond this discussion of nuclear structure. The parameter is important in the theory of pionic interactions, since the pion vertex has a $\sigma\tau$ factor. The strength of this interaction is crucial for predictions of a pion condensation phase transition[†]. The existence of an observable phase transition demands that G'_0 be as small as possible, certainly less than 1.2. Thus the Gamow-Teller energetics warn us not to have high expectations in seeking pion condensates. The parameter also comes into the theory of elastic pion scattering. McManus, et al.²⁴), in their fits to elastic scattering find that G'_0 is in the range 0.5 - 1.0, somewhat less than our value, and more optimistic for the phase transition. There is no real inconsistency between these values because the scattering requires the interaction at high momentum transfer, and the Gamow-Teller strength depends on the interaction at low momentum transfer. The various sources of information on the spin-isospin interaction are summarized in Table III, including some analyses that I have no time to mention.

Table III. Comparison of strengths for the $\sigma\tau$ interaction

<u>Phenomenological</u>	<u>$V_{\sigma\tau}$</u>
Giant Gamow-Teller State	160-240
Magnetic Transitions in Pb region (ref. 21)	275
π -Nucleus elastic scattering (ref. 24)	130-260
Magnetic Form Factor in ^{12}C (ref. 31)	220
Low Energy (p,n) Reaction (ref. 38)	191
<u>Theoretical</u>	
Reid potential in ^{90}Zr	170
Reid potential (ref. 23)	200
High-Brow Meson exchange (ref. 32)	230-370

We can also reexamine the M1 energetics with the Landau parameterization. In ^{208}Pb , the M1 strength has isovector character and we can just apply G'_0 . Using the same value as fits the Gamow-Teller energy, and the empirical particle-hole energies, I calculate

[†]In many theoretical papers on pionic interactions with nuclei, a similar symbol, g' , is used for a different unit of interaction strength,

$$V_{\text{eff}} = g' (390 \text{ MeV}\cdot\text{fm}^3) \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \delta(r_1 - r_2)$$

an excitation energy in ^{208}Pb of 7.8 MeV. With the Becchetti-Greenlees spin-orbit potential, the energy is somewhat higher. Thus this analysis opens the door again to the question of whether there is missed low-lying M1 strength in ^{208}Pb . In ^{48}Ca , it is not enough to know G'_0 to predict the M1 state. The state is a pure neutron excitation, which requires knowledge of the isoscalar parameter G_0 as well. We could use the data to determine the parameter (and find that $G_0 \approx 0$), but this is perhaps pushing the empirical analysis too far.

Strength

I now want to discuss the strength of the Gamow-Teller state. There are two aspects we can discuss, the relative magnitude of the upper and lower components, and the absolute cross section. The relative magnitudes can be understood with simple theory. Recall that the residual interaction shifts most of the strength into the upper state in the lp-lh model. The ratio of the strengths may be easily calculated from the eigenvectors associated with the empirical G'_0 interaction. In Table IV this theoretical ratio is compared with experiment. Typically we see a 4:1 ratio, with good agreement between theory and experiment.

Table IV. Relative strengths of giant Gamow-Teller and low-lying 1^+ excitations

<u>Nucleus</u>	<u>Theory</u>	<u>Experiment</u>
^{48}Ca	5	6 (ref. 2)
^{90}Zr	5	4 (ref. 25)
^{208}Pb	4	

The absolute magnitude of the cross section requires knowledge of the projectile interaction and the wave distortion effects in the target. Goodman, et al.²⁶⁾ have calibrated the reaction by comparing the cross sections to states with known Gamow-Teller strength. They arrive at a simple formula for the (p,n) reaction at 120 MeV,

$$\left. \frac{d\sigma}{d\Omega} \right|_0 = \left(\frac{A}{A+1} \right)^2 (8.3 \text{ mb/sr}) (N^D) \langle \sigma \tau_- \rangle^2 \quad (3)$$

Here N^D is the effect of the wave distortion and is given by

$$N^D = 1.74 e^{-.541 A^{1/3}} \quad (4)$$

In Table V we show how eq. (2) compares with the data. First notice that this formula works even on the p+n reaction: $\langle \sigma \tau_- \rangle^2 = 6$ for $p \rightarrow n$; $N^D = 1$, $A = 1$, and $\left. \frac{d\sigma}{d\Omega} \right|_0 \approx 12 \text{ mb/sr}$. In the heavy nuclei the cross section is substantially less than theory. But this discrepancy is also well known from the matrix elements of the β decay operator. The nuclei, such as ^{39}Ca and ^{41}Sc , which should have the full shell

Table V. Cross section for excitation of the giant Gamow-Teller state by the (p,n) reaction at 0°. The light nuclei are shown for calibration purposes (ref. 18). In the heavy nuclei, the theoretical Gamow-Teller strength is that of the upper state from the TDA calculation. The cross section is obtained from the strength with eqs. (3-4).

<u>Target</u>	<u>Theoretical</u> <u>$\langle \sigma_{\tau} \rangle^2$</u>	<u>Theoretical</u> <u>Cross Section</u> (mb/sr)	<u>Measured</u> <u>Cross Section</u> (mb/sr)
neutron	6	12.5	11
^{12}C	1.88	6.7	6
^{48}Ca	40	78.3	30
^{90}Zr	50	63.9	21
^{208}Pb	175	100	40

model strength in their beta decay, are actually retarded by a factor of two²⁷⁾. This may be partially understood in terms of mesonic effects²⁸⁾. The nuclear configuration space should be extended to include Δ -particles as well as nucleons. Instead of the independent particle strength function from Fig. 6, we should consider the one in Fig. 7. The Δ peak is larger because all of the nucleons can

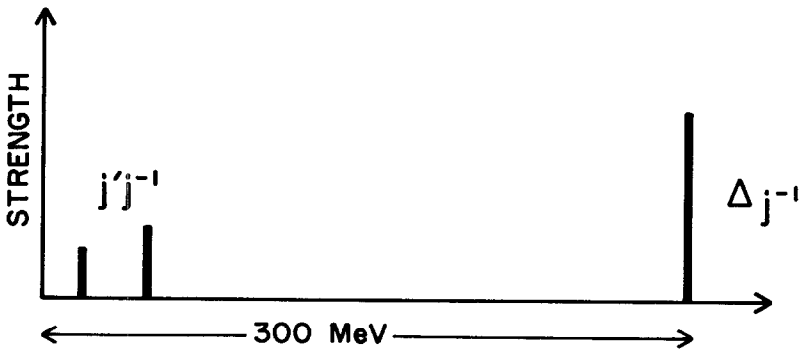


Fig. 7 The independent particle σ_{τ} strength function, including the Δ state of the nucleon.

participate in the Δ excitation, while the Pauli principle blocks most of them from the particle-hole excitation. It might seem that the states are too far away to be significant, but the pion couples more strongly to the Δ than to the nucleon. Quantitatively, the Δ states seem to account for about half of the observed quenching²⁸⁾. A similar story can be made for mesonic corrections to the M1 strength. The Δ produces a quenching by a factor of two in infinite nuclear matter, but is not so effective in finite nuclei²⁹⁾.

Width

In all of the heavier nuclei the width of the Gamow-Teller state is about 4 MeV. Since particle decay widths rarely exceed 1 MeV, we may assume that the width is due to mixing with more complicated states. We saw in the case of Gaarde's calculation of 2p-2h admixtures in ^{48}Ca , that at least qualitatively theory produces the width. One important point can be made without doing a complicated calculation. Namely the width is much smaller than a single-particle state would have with the same available energy. The widths of relevant single-particle states have been measured by Gales, et al.³⁰⁾. The $h_{11/2}$ hole state in ^{207}Tl is found at an excitation energy of 8 MeV and a width about 3.7 MeV. In comparison, the Gamow-Teller state has twice the excitation energy with the same width. There is clearly much more phase space available, even considering that the energy must be shared between a particle and a hole. I believe the reason for the relatively modest width is that there is an interference between the particle and the hole damping. This effect is significant in the damping of other giant vibrations³³⁾. It might be surprising that there should be such interference; the particle and hole have different spin and isospin projection, and so might be expected to couple differently to the core. However, according to calculation the most important core excitations for the damping are the low-lying spin-isospin scalar vibrations³⁴⁾, which do not couple to the valence particle spin or isospin. Thus the observation of this 4 MeV width seems to confirm theoretical ideas on the general mechanism of the damping. But more detailed work should be done on this subject.

The width of the Gamow-Teller strength function is an important ingredient in present day calculations of stellar nucleosynthesis. Calculation of beta-decay lifetimes over a broad range of the periodic table requires a suitably parameterized Gamow-Teller strength function. This is done in the gross theory of beta decay by Takahashi, et al.³⁵⁾, who describe the Gamow-Teller strength with a two-parameter function. The function is characterized by its mean energy and width. The energy is taken degenerate with the isobaric analog, which we now know is somewhat low. Takahashi, et al. find that a Gaussian r.m.s. width of 5 MeV gives a global fit to the data. We should express this as a full width at half maximum to compare with the measured Γ . For a Gaussian the relation is

$$\Gamma = 2.35 \sigma \quad (5)$$

Thus the gross model of beta decay uses a $\Gamma \approx 10$ MeV, much larger than the measured 4 MeV. This disagreement is surprising, even though we know that the beta decay is just sensitive to the low energy tail of the Gamow-Teller strength function. Perhaps the primary division of the strength functions into two pieces make the analysis by a single Gaussian misleading. It would be interesting to see the β -decay analysis redone in the light of the recent experimental results on the strength function³⁹⁾.

Higher Modes

There is good evidence that higher modes of spin excitation are visible in the intermediate energy (p,n) reaction. In this conference^{17,40)} there are several reports of a broad $L = 1$ bump, seen in nuclei ranging from ^{90}Zr to ^{208}Pb . The excitation energy is about

20 MeV and the width is about 10 MeV. Since we have the residual interaction from the energetics of the Gamow-Teller state, we can now calculate the $L=1$ states and compare with the experiments. In ^{90}Zr , the orbital excitation energy is ≈ 9 MeV for a Woods-Saxon potential, and is somewhat larger for a Hartree-Fock potential.

Also raising the excitation energy of $L=1$ states is a stronger residual interaction. This is because more particles can participate in the $L=1$ collective motion than in the $L=0$. In ^{90}Zr , the residual interaction is calculated to be nearly twice as strong for the $L=1$ states as for the Gamow-Teller state.

The spin orbit energy is less important in the $L=1$ state than in the Gamow-Teller state. I calculate its average at 1.6 MeV for the $L=1$, compared with 6 MeV for the Gamow-Teller state. Putting these contributions to the energy together, I find for the average energy

$$9 + 5.4 + 1.6 = 16 \text{ MeV,}$$

using the Woods-Saxon orbital energy. This is lower than experiment, but could easily be brought into agreement by using Hartree-Fock energies. For the systematics of the energy with mass number A , we can expect the orbital contribution to decrease as

$A^{-1/3}$, and the interaction energy to increase with the neutron excess. The overall dependence on A will then be much weaker than the typical $A^{-1/3}$ behavior of giant resonances. This is in accord with the experimental finding that the excitation is roughly constant from ^{90}Zr to ^{208}Pb .

The spin-orbit energy depends strongly on the J -coupling of the state. The 0^- state can only be made by spin flip, with configurations such as $h_{9/2}g_{9/2}^{-1}$. This state has the full spin-orbit energy of ≈ 6 MeV. On the other hand, in the 2^- state, configurations differing by two units of j are dominant, and the spin orbit energy is actually negative. For experiments that do not resolve the separate J -states, the effect of this will be to make the state appear broader. For ^{90}Zr , I calculate the r.m.s. energy spread of the three J -coupled states to be 4 MeV. The experimental width is quoted at ≈ 10 MeV, but this is the full width at half maximum of a Gaussian. Using eq. (5) for the relation of these two measures of the width, we see that our σ of 4 MeV implies $\Gamma=10$, in agreement with experiment. This provides a possible explanation of why the $L=1$ strength is so broad compared to the Gamow-Teller strength.

Conclusion

With the new experimental data, we understand much better now the energetics of the spin excitations. In broad outline the energies may be understood with a single interaction parameter G'_0 , but much work needs to be done to refine the description. The energies are sensitive to the velocity dependent parts of the interaction, parameterized by F_1 , F_2 , etc. Kohno and Ando⁹⁾ have shown how these parameters can be included in a more precise theory, for the case of spin-independent vibrations. The formula for the isovector giant states in the large A limit has the structure⁹⁾

$$\left(\frac{E}{E_0}\right)^2 = \frac{(1 + F'_1/3)}{(1 + F_1/3)} \left(\frac{9}{15} + \frac{F'_0}{3} + \theta(z^2)\right) z^2$$

where z is the root of a spherical Bessel function and E_0 is determined from the ground state size and kinetic energy density. Except for the spin orbit energies, analogous formulas should apply to spin-flip excitations, by changing F'_1 to G'_1 . Thus in principle a more refined treatment of the $L=1$ energies can be used to extract G'_1 .

The strength of the spin excitations is still a problem for theorists. The data is compelling showing these excitations to be suppressed. There are clear mechanisms in meson theory for the suppression, but as a quantitative matter the mesonic effects do not appear strong enough.

Acknowledgment

The author acknowledges discussions with H. Toki and S. M. Austin. This work was supported by the National Science Foundation under grant no. PHY-79-22054.

References

1. W. Steffen, et al., Darmstadt preprint IKDA 80/5 (1980).
2. B.D. Anderson, J. Knudson, P.C. Tandy, et al., Kent State Preprint (1980).
3. G. Bertsch, *The Practitioner's Shell Model*, (North-Holland, 1972) p. 80.
4. R. Holt and H. Jackson, *Phys. Rev. Lett.* 36 (1976) 244.
5. W. Knüpfner, et al., *Phys. Lett.* 77B (1978) 367.
6. D.J. Horen, et al., *Phys. Lett.* 79B (1978) 39.
7. R.J. Holt, et al., *Phys. Rev.* C20 (1979) 93.
8. F. Becchetti and G. Greenlees, *Phys. Rev.* 182 (1969) 1193.
9. M. Kohno and K. Andō, *Prog. Theo. Physics* 61 (1979) 1065; K. Andō, conference contribution, p. 192.
10. W. Sterrenburg, et al., conference contribution, p. 176.
11. S. Cohen and D. Kurath, *Nucl. Phys.* 73 (1965) 1.
12. G.E. Brown, et al., *Nucl. Phys.* A330 (1979) 290.
13. R. Scheerbaum, *Phys. Lett.* 61B (1976) 151.
14. T.W. Donnelly and J.D. Walecka, *Ann. Rev. Nucl. Sci.* 25 (1975) 329.
15. R. Doering, et al., *Phys. Rev. Lett.* 35 (1975) 1691.
16. P. Marmier and E. Sheldon, in *Physics Of Nuclei and Particles*, vol. II (Academic Press, 1970) p. 1079.
17. D.J. Horen, et al., conference contribution, p. 272.
18. K. Ikeda, S. Fujii and J.I. Fujita, *Phys. Lett.* 3 (1963) 271; see also H. Ejiri and J.I. Fujita, *Phys. Rep.* 38 (1978) 85.
19. C. Gaarde, et al., *Nucl. Phys.* A334 (1980) 248.
20. C. Gaarde, private communication.
21. P. Ring, et al., *Nucl. Phys.* A206 (1973) 105.
22. P. Ring and J. Speth, *Nucl. Phys.* A235 (1974) 315.
23. S.O. Bäckman, et al., *Nucl. Phys.* A321 (1979) 10.
24. K. Stricker, J.A. Carr, H. McManus, *Phys. Rev. C*, to be published.
25. T. Nees, private communication.
26. C.D. Goodman, et al., *Phys. Rev. Lett.* 44 (1980) 1755.
27. S. Raman, et al., *Atomic Data and Nuclear Data Tables* 21 (1978) 567.
28. E. Oset and M. Rho, *Phys. Rev. Lett.* 42 (1979) 47.

29. H. Toki and W. Weise, Regensburg preprint (1980).
30. S. Gales, et al., Phys. Rev. C18 (1978) 2475.
31. H. Toki and W. Weise, Phys. Lett. 92B (1980) 265.
32. G.E. Brown, et al., Nucl. Phys. A286 (1977) 191.
33. P.F. Bortignon, conference contribution, p. 183.
34. G.F. Bertsch, et al., Phys. Lett. 80B (1979) 161.
35. K. Takahashi, M. Yamada, T. Kondoh, Atomic Data and Nuclear Data Tables 12 (1973) 101.
36. A. Arima and H. Hyuga, in Mesons in Nuclei, ed. by M. Rho and D. Wilkinson (North-Holland, 1979) p. 685.
37. D.E. Bainum, et al., Phys. Rev. Lett. 44, (1980) 1751.
38. S.M. Austin, in The (p,n) Reaction and the Nucleon-Nucleon Force, ed. C. Goodman, et al., (Plenum, 1980).
39. T. Oda and H.V. Klapdor, conference contribution, p. 278.
40. W. Sterrenburg, et al., conference contribution, p. 175.
E. Sugarbaker, et al., conference contribution, p. 207.
C.D. Goodman, et al., conference contribution, p. 241.