The Duflo-Zuker mass formula–what is it?

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SHELL OCCUPANCY AND DEGENERACY VARIABLES

The program uses several grouping of shells. The harmonic oscillator major shell is divided into a subshell consisting of the maximum-j spherical shell and a subshell containing all the others. The formula requires the occupation numbers and degeneracies of the shells in various groupings:

degeneracy	occupation number	kind of shell	fortran variable	sequence
d_m	n_m	subshell	`NOC`	2,0,4,2,6,6,...
d_h	n_h	h.o.	`DEGI`	2,6,12,...
d_s	n_s	spin-orbit	`DEI,OEI`	2,4,8,14,...

As examples,in the *sd* major shell the $d_{5/2}$ with a degeneracy of 6 is the lower subshell and the set $d_{3/2}, s_{1/2}$ with a degeneracy of 6 is the upper. The spin-orbit grouping puts the $d_{5/2}$ with the $p_{1/2}$ below and the subshell $d_{3/2}, s_{1/2}$ with the $f_{7/2}$ above. Note that for the $N = 0$ shell the upper subshell has a degeneracy of zero (but is still formally included in the sums).

INDIVIDUAL TERMS

The numbers refer to coefficients of the parameters `A(I)` in the program.
1. Define a "density"

$$\rho = \sum_i n_h(i)/\sqrt{d_h(i)}. \tag{1}$$

The first term is a constant times

$$(\rho_n^2 + \rho_p^2)/A^{1/3}$$

where the subscript distinguishes neutrons and protons.
3. This term defines a spin-orbit density

$$\rho_{so} = \sum_i n_m(i) v_{so}(i).$$

The weight function v_{so} is such that rho_{so} which is maximum if the Fermi level is at the spin-orbit shell closure and is zero if it is at the harmonic oscillator shell closure. The contribution to the energy is proportional to ρ_{so}^2.
5. ?
7. This term is linear in ρ_{so}, and thus corresponds most closely to the spin-orbit energy as we know it. The energy is given by

$$\left(\sum_i n_m(i) v_{so}(i)/\sqrt{d_h(i)}\right)\left(\sum_i n_m(i)/\sqrt{d_h(i)}\right)$$

9.

$$\sum_i n_h^2(i)/d_h(i)$$

11.?
15. The next few terms are constructed from a quantity that could be called the polarizability of the valence spin-orbit shell,

$$\rho_q = n_s(d_s - n_s)/d_s$$

In term 15, the combination is called "S3" and is given by

$$\rho_q(2n_s - d_s)/A^{-1/3}$$

17. This is the combination "qqm" which behaves as a neutron-proton interaction:

$$\rho_{qn}\rho_{qp}$$

19.-25. Apply to deformed nuclei only.
27. Symmetry energy as in the Weizäcker formula.
2n. The even-numbered term 2n. (up to 30.) is the same as 2n-1. except for an additional factor of $A^{-1/3}$.
31. The Coulomb energy as in the Weizäcker formula.
32.,33. The pairing energy.

NOTES

1. Term (1) is very nearly equal to $Z + N$ for medium and heavy nuclei (see the program a_scale.f90). Thus terms (1) and (2) function as the volume and surface energies in the Weizsäcker formula.
2. The parameter count should be increased by several. Search on "0.76","0.25", and "16." in the program.
3. Turning off the special treatment of deformed nuclei, the r.m.s. error increases to 1.5 MeV, only a factor 2 better than the 5-parameter Weizäcker formula.