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ectric dipole moment to achieve hitherto unattainable levels of sensitiv- ; worth making, here, the perhaps obvious remark that the stability of gas BEC against thermal disorder is usually not simply a consequence of l factors but also of energetic ones, in that for repulsive interactions (the ase) the "Fock" term in the interaction energy positively advantages the te.

'superfluid amplification" property has some very intriguing consequences nterpretation of the notion of "randomness" and the related concept of ement" in quantum mechanics. Crudely speaking, in a normal (uncon- system random forces (noise) will act independently on the different atoms any-body system, and because any measurement, even a "single-shot" one, lve averaging over the behavior of the N atoms, the effects of the noise visible even on a single run. For a BEC system, on the other hand, any " effects will be *the same* for all atoms of the condensate, and the sta- analysis of experiments must take this into account. As an example let's the diffusion of the relative phase of two different hyperfine species as in Ramsey-fringe experiments. One needs to distinguish between two types those (such as recombination) which leave the relative phase definite on n run but random from run to run (so that an appropriately defined (com- mber which one might call the "degree of phase coherence" is large for each al run but when averaged over runs gives a small or zero value) and those the nonlinear effect of the mean field) which genuinely decrease the degree coherence on each individual run. It follows that a mechanical calculation jection of the single-particle density matrix, such as one is used to doing ondensed systems, may give a very misleading picture of the actual experi- behavior–a point which is, of course, by now well appreciated in the related of the famous MIT interference experiment. While in these particular cases e we can by now claim a reasonable degree of understanding, there are a ost of related problems (e.g., those connected with the initial formation of densate and with various types of nonlinear damping) where I believe our ation of the interplay between what one might call the effects of "classical" uantum" uncertainty is still at a very rudimentary stage.

; brings me back to Eugene Feenberg; for I believe that one attitude that I share in our approach to many-body physics is a profound respect for the nger wave function and a certain wariness about other, more "fancy" de- ns. The situation discussed in the last paragraph is one example of a case in proper Schrödinger description of the BEC system helps to avoid concep- falls, but there are many others; I would myself particularly cite the dangers ted with an insufficiently careful use of the concept of "spontaneously bro- 1) symmetry". I believe that Eugene, were he alive to-day, would revel in ellectual challenges posed by the BEC alkali gases, and it is a pleasure to e this brief note to his memory.

RPMBT-10 Challenge Competition Winning Entry

THE MBX CHALLENGE COMPETITION: A NEUTRON MATTER MODEL

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In this paper I report my solution to MBX Challenge Competition. Namely, the Bertsch, nonparametric model of neutron matter is analyzed and strong indications are found that, in the infinite system limit, the ground state is a Fermi liquid with an effective mass.

1 Introduction

As a challenge to the participants of the Tenth International Conference on Recent Progress in Many-Body Theory, G. F. Bertsch[1] proposed the following problem. It is:

What are the ground state properties of the many-body system composed of spin-1/2 Fermions interacting via a zero-range, infinite scattering-length contact interaction.

It may be assumed that the interaction has no two-body bound states. Also, the zero range is approached with finite-ranged forces and finite particle number by first taking the range to zero and then the particle number to infinity.

This problem is tricky in the following sense, if one reverses the limit order and takes the particle number to infinity before the range goes to zero, one obtains the well-known nuclear collapse result where the whole system collapses into a region of the order of the range of the potential in size. Likewise, if the particles were Bosons, collapse would occur. A fuller exposition of the solution may be found in Ref. 2.

2 Methods

How shall we solve this problem?

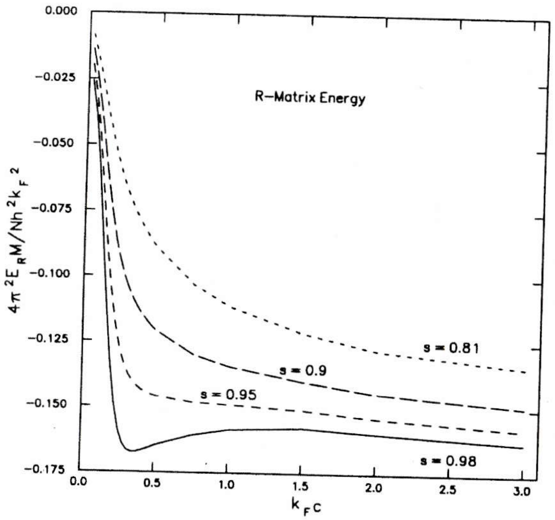
We will use a combination of two types of series expansions.

1. An expansion of the ground state energy in powers of the potential strength for fixed density.

2. A low density expansion of the ground state energy for fixed potential strength.

For ease of exposition, I will use the square-well potential,

$$V(r) = \begin{cases} -V_0, & \text{if } r < c, \\ 0, & \text{if } r > c. \end{cases}$$



ıre 1. The numerical evaluation of the R-matrix energy. The short dashed curve is for $s = 0.81$, long dashed curve is for $s = 0.9$, the dashed curve is for $s = 0.95$, and the solid curve is for 0.98.

this potential, the strength is

$$s = \frac{4}{\pi^2}\frac{MV_0}{\hbar^2}c^2.$$

for our problem we want $s = 1$. The potential energy expansion is

$$\frac{E}{N} = \frac{3\hbar^2 k_F^2}{10M} + \frac{\pi^2\hbar^2}{4Mc^2}A_1 s + \frac{\pi^4\hbar^2}{16Mc^2}A_2 s^2 + \dots,$$

he first term for neutrons is:

$$A_1 = \frac{3}{4\pi\kappa_F^3 V_0}\int_{|\vec\mu|\le\kappa_F,\,|\vec\nu|\le\kappa_F} d\vec\mu\, d\vec\nu\left[\tilde v(0) - \frac{1}{2}\tilde v(|\vec\mu - \vec\nu|)\right],$$

erms of the dimensionless variables,

$$\vec\rho = \vec r/c, \quad \vec\kappa = c\vec k, \quad c^3\tilde v(\kappa) = \frac{1}{(2\pi)^3}\int d\vec r\, V(r)\exp(-i\vec k\cdot\vec r).$$

can be worked out exactly and $\tilde v(0) = -1/(6\pi^2)$. Since we are concerned with imit as $c \to 0$, and finite Fermi momentum, $\kappa_F \to 0$. Thus we get,

$$A_1 = -\frac{\pi}{3}\kappa_F^3.$$

he time being, we will hold the scattering length fixed and finite, and let $c \to 0$. potential strength stays finite, but the potential depth becomes infinite. The

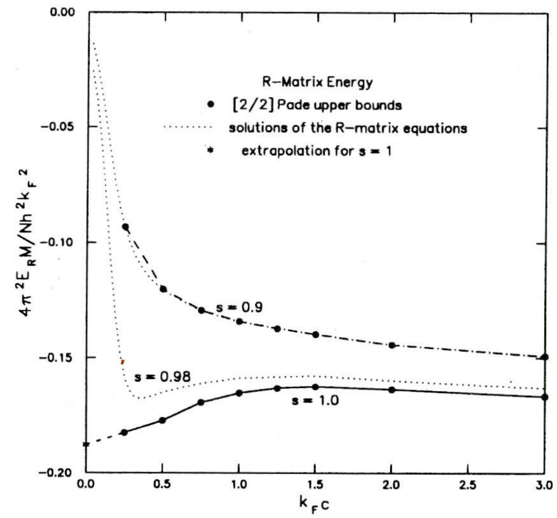


Figure 2. The Padé approximant upper bounds on the R-matrix-approximation energy divided by $\hbar^2 k_F^2/M$ for various potential strengths. Some of the numerical solutions of the R-matrix equation are included for reference.

standard way to deal with this situation is to put ladder insertions in all the higher order terms. Skipping the details, we have for low-density

$$\frac{EM}{N\hbar^2} = k_F^2\left[\frac{3}{10} + \frac{1}{3\pi}k_F a + 0.055661(k_F a)^2 \right.$$
$$\left. +0.00914(k_F a)^3 - 0.018604(k_F a)^4 + o(k_F^4)\right].$$

which just depends on the scattering length a and not on the shape of the potential. The case of interest is, of course, given by the limit as $a \to \infty$. Before considering the limit $a \to \infty$, our approach is to take some guidance from the low density expansion. Usually one would start with the K-matrix, however in the case of a purely attractive potential, it is plagued[3] with "Emery Singularities." Consequently, I will use the R-matrix[4] formulation. The difference between the K-matrix in ladder approximation and the R-matrix is in the Green's function. For the K-matrix the Green's function is

$$G_{k,l}(r,r') = \int_0^\infty \frac{k''^2 j_l(k''r)j_l(k''r')}{k''^2 - k^2}F(p,k'')\,dk'',$$

where F reflects the Pauli priniciple. It has been shown that,

$$K_l(k) = \frac{R_l(k)}{1 + (\frac{1}{2}r_1 - k^2/k_F)R_l(k)},$$

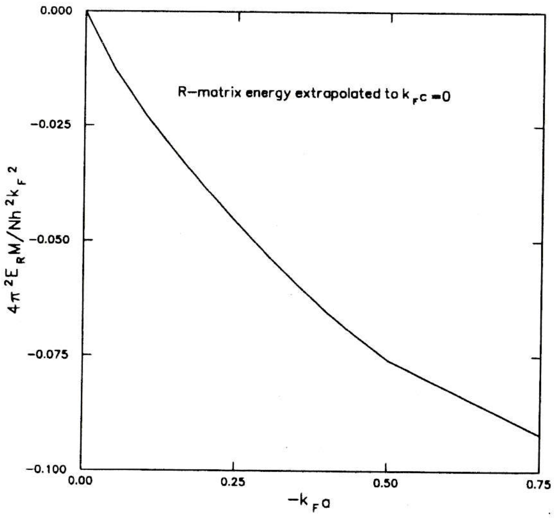


Figure 3. The extrapolation of the R-matrix energy to $k_F c = 0$ as a function of $k_F a$.

where

$$\tau_1 = (k_F p)^{-1}\{(k^2 + \frac{1}{4}p^2 - k_F^2)$$
$$\times \ln[(k_F^2 + k_F p + \frac{1}{4}p^2 - k^2)/(k_F^2 - \frac{1}{4}p^2 - k^2)]$$
$$+ \left(1 - \frac{p^2}{4k_F^2}\right) \ln[(k_F + \frac{1}{2}p)/(k_F - \frac{1}{2}p)]\}$$
$$+ \left(\frac{k}{k_F}\right) \ln[(k_F + \frac{1}{2}p + k)/(k_F + \frac{1}{2}p - k)].$$

τ_1 is lower semi-bounded, but diverges logarithmically to plus infinity. However, when it is negative, there is a singularity in the K-matrix. One consequence of this result is that although it is expected that the radius of convergence of R in powers of the strength s is unity, the radius of convergence of the K-matrix series is zero.

In Fig. 1 we see the numerical results of the evaluation of the R-matrix energy. Notice that outside a small initial region, these curves are relatively flat.

2.1 Method 1

A series expansion in the potential strength can be computed numerically for the R-matrix. An examination of its structure shows it to be that of a two-side (or Hamburger) moment problem. It has been shown that for this case, inside the radius of convergence, that all Padé approximants[5] form upper bounds.

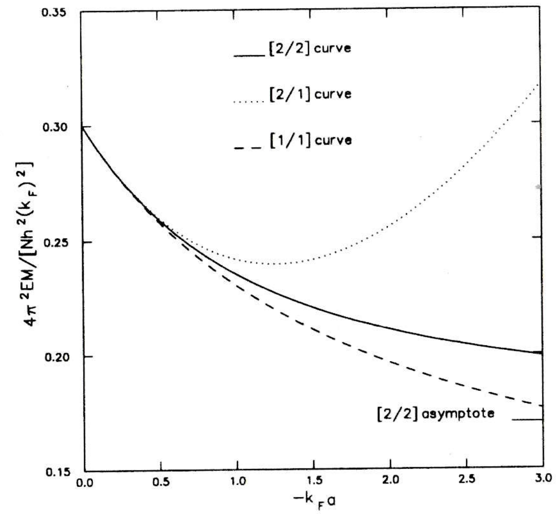


Figure 4. The ratio of the many-body energy per particle to $\hbar^2 k_F^2/M$, verses $-ak_F$. For of interest, $a \to -\infty$ is expected.

These numerical results are displayed in Fig. 2. The value sought, is the olation to $k_F c = 0$ which is about $-0.18\hbar^2 k_F^2/M$.

2.2 Method 2

By adjusting the potential strength and k_F we can compute the behavio R-matrix energy as $k_F c \to 0$ for fixed $k_F a$. In Fig. 3 there is a plot of the

Here we need to extrapolate this curve to $k_F a = -\infty$. At low Fermi mo the leading coefficient should be $1/(3\pi)$ so our extrapolation is about 2% l This behavior is not inconsistent with the results of the previous plot.

A bit of additional information is that the asymptote for the [2/2] Padé imant to the ladder energy is about $0.24\hbar^2 k_F^2/M$ which is not vastly differ our estimates for the R-matrix energy, and also corresponds to no negativ ground state.

We are now in a position to apply method 2 to the complete energy. compute various Padé approximants to the low density expansion, yiel results displayed in Fig. 4.

Numerically, the asymptote for the [2/2] is $0.1705\hbar^2 k_F^2/M$. The va responds to a shift in the complete energy from the ideal gas energy o $-0.1295\hbar^2 k_F^2/M$.

If we now apply method 1 to previously computed data the best Padé mant is the [3/1]. These results are shown in Fig. 5.

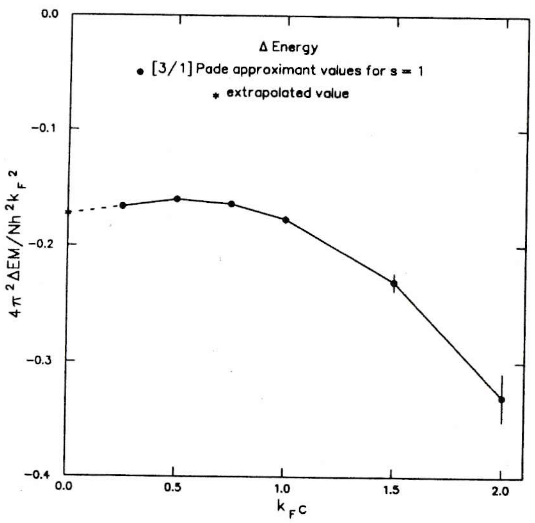


Figure 5. The estimates of the many-body energy per particle based on the series expansions in potential strength. The extrapolation to $k_F c = 0$ is also shown. The error bars reflect only the coefficient uncertainty.

The result of this calculation is about $\Delta E = -0.17\hbar^2 k_F^2/M$. All together, I estimate that this model of the interactions in neutron matter gives $\Delta E = -(0.17 \pm 0.04)\hbar^2 k_F^2/M$.

3 Conclusions

The reasonable concordence of both methods for the computation of the ground-state energy means that the ground state of system behaves like that of a Fermi liquid, with an effective mass of $(2.3 \pm 0.5)M$. The wave-function is expected to correspond to that structure, aside from a set of exceptional points where $\vec{r}_i = \vec{r}_j$, the origins of the set of relative coordinates between all the pairs.

Acknowledgements

I wish to thank Prof. M. DeLlano for drawing Ref. 6 to my attention. It contains a nice survey of the properties of the contact (delta function) interaction in various dimensions.

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