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DAMPING OF NUCLEAR EXCITATIONS AT FINITE TEMPERATURE

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Abstract: We calculate the damping of single-particle motion and of vibrational motion to lowest order in the coupling between the particles and the vibrations, using the finite temperature Matsubara formalism. The derived formulas have a complicated structure which however can be mostly understood in physical terms. We apply the theory to single-particle states in heavy nuclei, to the giant dipole vibration in ^{90}Zr, and to the giant quadrupole vibration in ^{208}Pb. Even at temperatures of the order of 3 MeV the main peak of the giant vibrations remains essentially unaffected although it acquires a long tail at the low-energy end.

1. Introduction

It has recently become possible to measure the properties of nuclei at very high excitation energy by means of heavy-ion collisions. In particular, the γ-decay of these excited nuclei, including the observation of the giant dipole transition, have been measured using large arrays of γ-detectors [cf. refs. [1-6]) and references therein]. The theory for such processes is necessarily statistical in nature.

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In what follows we investigate the consequences of finite temperature on the damping width of single particles and of giant resonances in terms of standard Green function theory. Although we are interested mostly in the response of the system, which directly gives the electromagnetic transition properties, we also find it necessary to treat the finite temperature theory of the single-particle Green function.

The previous work on finite Green function theory in nuclear physics has been limited. Most applications of finite temperature theory have been treated at the level of mean field theory. Dealing with dynamic questions, a formula for the damping of particle motion in infinite fermion systems was derived by Morel and Nozières [7]). For the vibrational motion only Landau damping is permitted in mean field theory, and this turns out not to be an important damping mechanism or one that varies much with temperture (cf. also ref. [8]) and refs. therein). Absorption of oscillations in a Fermi liquid has been discussed by Landau [9]).

The physical assumptions on which the present work is based are that Hartree-Fock theory of the single-particle motion and RPA theory of vibrations provide a usable lowest order approximation for nuclei*, and that the main damping mechanisms arise from the coupling of particles to the low-energy surface vibrations. Our work follows the assumptions and methods we applied to damping questions at zero temperature [11]), also discussed in a review [12]). A key element of that work is a self-consistency requirement which fixes the strength of the coupling, allowing the calculated damping to be quite insensitive to other details of the assumed hamiltonian.

This article is organized as follows. In the next section we present the formulas for the damping, derived in the Matsubara finite temperature formalism. These formulas are quite complicated in structure, but they can be cast in a form in which the most important terms have a simple physical interpretation. In the third section, we evaluate the formulas for a range of nuclei and temperatures, finding the result that giant resonances are remarkably stable, even for temperatures of the order of magnitude of the lowest vibrational energies.

2. Formulation of the model

Our finite temperature dynamics is based on many-particle Green function theory in the Matsubara formalism. We refer the reader to ref. [13]) for the graphical rules of the formalism. The rules are the same as for zero temperature Green functions except that the integrals over frequency are replaced by sums over discrete imaginary frequencies. A real-time Green function or response is found by analytic continuation in the frequency variable. The Green function theory has the advantage that we are assured that the quantities, like e.g. the mass operator which is the central subject of the present paper, are evaluated consistently to a given order of perturbation theory in the grand canonical ensemble. However, the expression for the response

* See also, e.g., ref. [10]).

to second order in the residual interaction is rather involved, containing terms with apparent singularities that cancel in the total expression. Because we could not find in the literature the explicit formulas, we present the detailed expressions here.

We first discuss the single-particle Green function. The first correction to the unperturbed Green function is the self-energy associated with the graph of fig. 1, which is given by [cf. eqs. (3.4.14), (3.5.1) and (3.5.5), Mahan [13])]

$$(\Sigma^{(1)}(1, ip_n))_p = -T \sum_{2,\lambda} V^2(1,2;\lambda) \sum_{i\omega_n} \frac{1}{i(p_n - \omega_n) - \varepsilon_2} \left(\frac{1}{i\omega_n - \omega_\lambda} - \frac{1}{i\omega_n + \omega_\lambda} \right), \quad (1)$$

where i labels a particle state with energy ε_i, λ labels an RPA vibration with energy ω_λ, and $V(i,j;\lambda)$ is the particle-vibration coupling strength. The temperature is T and the boson frequency sum is over $\omega_n = 2\pi n$ with n an integer. The explicit linear dependence with the temperature is cancelled by the boson sum $\sum_{i\omega_B} \sim 1/T$. This frequency sum is performed as described in the textbooks to give the following formula for the self-energy [cf. eqs. (3.51) and (3.3.11) of Mahan [13])].

$$(\Sigma^{(1)}_{\text{ret}}(\varepsilon + iI))_p = \sum_{2,\lambda} V^2(1,2;\lambda) \left\{ \frac{1 + n_B(\lambda) - n_F(2)}{\varepsilon + iI - \varepsilon_2 - \omega_\lambda} + \frac{n_B(\lambda) + n_F(2)}{\varepsilon + iI - \varepsilon_2 + \omega_\lambda} \right\}. \quad (2)$$

Here $n_B(\lambda) = (\exp(\omega_\lambda / T) - 1)^{-1}$ and $n_F(i) = (\exp(\varepsilon_i / T) + 1)^{-1}$ are the Bose and Fermi occupation factors at finite temperature.

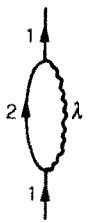

Fig. 1. Self-energy diagram for the single-particle state. An arrowed line describes the particle propagation while a wavy line describes the vibration.

The structure of eq. (2) may be easily understood. We examine the imaginary part of the expression within parentheses, which is nonvanishing only at energies corresponding to physical transitions,

$$\text{Im}\,(\Sigma^{(1)}_{\text{ret}}(\varepsilon + iI))_p \sim \{[1 + n_B(\lambda) - n_F(2)]\delta(\varepsilon - \varepsilon_2 - \omega_\lambda) + [(n_B(\lambda) + n_F(2)]\delta(\varepsilon - \varepsilon_2 + \omega_\lambda)\}. \quad (3)$$

Two terms are present because the intermediate state can be formed either by creation or annihilation of a vibrational quantum. The dependence on the occupation factors may be put in a more familiar form recognizing that the Green function of the Matsubara formalism includes both particle and hole propagation. In keeping

with the discussion above, a naive interpretation of the single graph in fig. 1, which also turns out to be correct, is that it contains the four time-ordered $T=0$ processes shown in fig. 2. The imaginary part of these graphs at finite temperature should include, besides the δ-function which ensures energy conservation, a thermal Pauli blocking factor for the particle (or hole) in the intermeditate state, and a matrix element for the creation or annihilation of the vibration, with the dependence on the number of quanta in the initial stae. The result for the 4 terms is given by

$$(1-n_F(2))(1+n_B(\lambda))\delta(\omega-(\varepsilon_2+\omega_\lambda))+n_F(2)(1+n_B(\lambda))\delta(\omega-(2\omega-\varepsilon_2+\omega_\lambda))$$
$$+(1-n_F(2))n_B(\lambda)\delta(\omega+\omega_\lambda-\varepsilon_2)+n_F(2)n_B(\lambda)\delta(\omega+\omega_\lambda-(2\omega-\varepsilon_2)), \qquad (4)$$

which is identical to eq. (3).

The appeance of boson occupation factors in the above expressions is a consequence of treating all vibrations as phonons, as done in the random phase approximation (RPA) approach. In ref. [10]) the correcting terms arising due to Pauli principle violations, and which imply boson occupation factors calculated at the particle-hole excitation energy, are discussed in detail. Similar to what we found at $T=0$, cf. ref. [11]), these corrections are also small, as discussed below.

For an infinite system, the behavior of the self-energy near the Fermi surface can be determined analytically by replacing the sums over states in eq. (2) by integrals over momenta. The resulting formula has a dependence on the energy $\omega=\varepsilon-\varepsilon_F$ of the particle above the Fermi energy ε_F and the temperature T given by*

$$\mathrm{Im}\,(\Sigma^{(1)}(T,\omega))_p \sim \{(\pi T)^2+\omega^2\}. \qquad (5)$$

The quadratic dependence on these variables is not at all realistic for surface-dominated systems [12,14]), but we shall find it useful to abstract from eq. (5) the idea that the exponent of the temperature dependent term is the same as that of the excitation energy, with a scaling factor (cf. appendix).

In fact, unlike the zero-temperature Green function formalism, a single-particle state may not be specified as a particle or hole externally; the grand canonical ensemble fixes the particle-hole character by the energy with respect to the Fermi

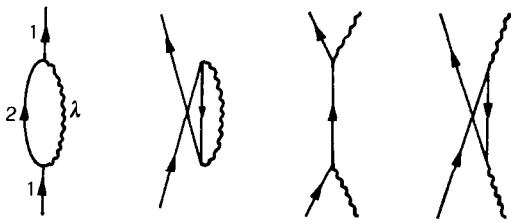

Fig. 2. The four time-ordered $T=0$ processes corresponding to the diagram of fig. 1, at $T\neq 0$.

* The formula of Morel and Nozières [7]) has the factor $(1+\exp(\omega/T))^{-1}$ and is not symmetric about the Fermi surface because their Green function does not include hole propagation.

energy. A consequence of this fact is that the zero temperature limit of the finite temperature formalism is not exactly the same as the zero-temperature theory. This is because single-particle states which are moved across the Fermi surface by the interaction will be treated differently in the two descriptions (cf. eq. (11) and following discussion).

We now discuss the vibration Green function. The lowest order correction to the unperturbed Green function is given by the processes shown in figs. 3 and 4. The frequency sum for the graphs 3a, 3b and 4 are given by

$$(P^{(2)}(\mathrm{GR}, i\omega_n))_{\mathrm{p}} = -T^2 \sum_{1,0} V^2(1,0;\mathrm{GR}) \sum_{2,\lambda} V^2(1,2;\lambda)$$
$$\times \sum_{ip_n} \frac{1}{(ip_n-\varepsilon_1)^2} \frac{1}{i(p_n-\omega_n)-\varepsilon_0} \sum_{i\omega_{n'}} \frac{1}{i(p_n-\omega_{n'})-\varepsilon_2} \left(\frac{1}{i\omega_{n'}-\omega_\lambda} - \frac{1}{i\omega_{n'}+\omega_\lambda} \right), \tag{6a}$$

$$(P^{(2)}(\mathrm{GR}, i\omega_n))_{\mathrm{h}} = -T^2 \sum_{1,0} V^2(1,0;\mathrm{GR}) \sum_{2,\lambda} V^2(1,2;\lambda)$$
$$\times \sum_{ip_n} \frac{1}{[i(p_n-\omega_n)-\varepsilon_1]^2} \frac{1}{(ip_n-\varepsilon_0)} \sum_{i\omega_{n'}} \frac{1}{i(p_n+\omega_{n'}-\omega_n)-\varepsilon_2} \left(\frac{1}{i\omega_{n'}-\omega_\lambda} - \frac{1}{i\omega_{n'}+\omega_\lambda} \right), \tag{6b}$$

and

$$(P^{(2)}(\mathrm{GR}, i\omega_n))_{\mathrm{v}} = T^2 \sum_{12,30,\lambda} V(1,0;\mathrm{GR}) V(2,3;\mathrm{GR}) V(12,30;\lambda)$$
$$\times \sum_{i\omega_{n'}} \sum_{ip_n} \left\{ \frac{1}{i(p_n+\omega_n)-\varepsilon_1} \frac{1}{i(p_n+\omega_n-\omega_{n'})-\varepsilon_2} \frac{1}{i(p_n-\omega_{n'})-\varepsilon_3} \right.$$
$$\left. \times \frac{1}{ip_n-\varepsilon_0} \left(\frac{1}{i\omega_{n'}-\omega_\lambda} - \frac{1}{i\omega_{n'}+\omega_\lambda} \right) \right\}, \tag{7}$$

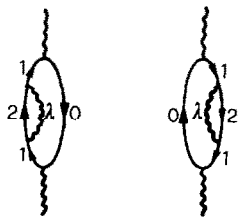

Fig. 3. Single-particle self-energy corrections to the RPA Green function.

Fig. 4. Lowest order vertex correction to the RPA Green function.

where $p_n = (2n+1)\pi$ while $\omega_{n'} = 2\pi n'$, with n and n' integers. Note the difference in overall minus sign between eq. (7) and eq. (3.4.16) of Mahan [13]). This is due to the fact that in this reference the sign of the matrix element $V(12, 30, \lambda)$ is included in the overall phase (rule 6, p. 156).

After performing the sums, making some algebraic manipulations, and analytically continuing the result from $i\omega_n$ to real ω, we arrive at the expression to be used in the actual calculations

$$
\begin{aligned}
(P^{(2)}_{\mathrm{ret}}(\mathrm{GR}, \omega + iI))_{\mathrm{p}} = \sum_{1,0} V^2(1,0;\mathrm{GR}) \sum_{2,\lambda} V^2(1,2;\lambda) \\
\times \left\{ \frac{n_{\mathrm{B}}(\lambda) + 1 - n_{\mathrm{F}}(2)}{\Delta\omega_{\lambda 20}} \left[\frac{n_{\mathrm{F}}(0) - n_{\mathrm{F}}(1)}{(\Delta\omega_{01})^2} + \frac{n_{\mathrm{F}}(1)}{(\Delta\varepsilon_{21})^2} \right] \right. \\
\left. - \frac{n_{\mathrm{B}}(\lambda) n_{\mathrm{F}}(2)}{\Delta\omega_{\lambda 20}(\Delta\varepsilon_{21})^2} + \frac{(n_{\mathrm{B}}(\lambda) + 1 - n_{\mathrm{F}}(2)) n'_{\mathrm{F}}(1)}{\Delta\omega_{01}\Delta\varepsilon_{21}} \right\} \\
- \{\text{same as above with } \omega_\lambda \to -\omega_\lambda,\ n_{\mathrm{B}}(\lambda) \to n_{\mathrm{B}}(-\lambda) \\
= -(1 + n_{\mathrm{B}}(\lambda))\}\,, \qquad (8a)
\end{aligned}
$$

$$
(P^{(2)}_{\mathrm{ret}}(\mathrm{GR}, \omega - iI))_{\mathrm{h}} = (P^{(2)}_{\mathrm{ret}}(\mathrm{GR}, -\omega - iI))_{p}\,, \qquad (8b)
$$

and

$$
\begin{aligned}
(P^{(2)}_{\mathrm{ret}}(\mathrm{GR}, \omega + iI))_{\mathrm{v}} = \sum_{12,30,\lambda} V(1,0;\mathrm{GR}) V(2,3;\mathrm{GR}) V(12,30;\lambda) \\
\times \left\{ n_{\mathrm{B}}(\lambda) \left[- \frac{n_{\mathrm{F}}(0)}{\Delta\omega_{01}\Delta\omega_{\lambda 20}\Delta\varepsilon_{30}} - \frac{n_{\mathrm{F}}(1)}{\Delta\omega_{01}\Delta\omega_{-\lambda 13}\Delta\varepsilon_{21}} \right. \right. \\
\left. + \frac{n_{\mathrm{F}}(2)}{\Delta\omega_{32}\Delta\omega_{\lambda 20}\Delta\varepsilon_{21}} + \frac{n_{\mathrm{F}}(3)}{\Delta\omega_{32}\Delta\omega_{-\lambda 13}\Delta\varepsilon_{30}} \right] \\
+ \frac{1}{\Delta\omega_{32}\Delta\omega_{01}} \left[- \frac{n_{\mathrm{F}}(1)[1 - n_{\mathrm{F}}(2)]}{\Delta\varepsilon_{21}} - \frac{n_{\mathrm{F}}(0)[1 - n_{\mathrm{F}}(3)]}{\Delta\varepsilon_{30}} \right. \\
\left. \left. - \frac{n_{\mathrm{F}}(0)[1 - n_{\mathrm{F}}(2)]}{\Delta\omega_{\lambda 20}} + \frac{n_{\mathrm{F}}(1)[1 - n_{\mathrm{F}}(3)]}{\Delta\omega_{-\lambda 13}} \right] \right\} \\
- \{\text{same as above with } \omega_\lambda \to \omega_\lambda, \\
n_{\mathrm{B}}(\lambda) \to n_{\mathrm{B}}(-\lambda) = -(1 + n_{\mathrm{B}}(\lambda))\} \qquad (9)
\end{aligned}
$$

$$
\Delta\omega_{\pm\lambda jj'} = \omega + iI - (\pm\omega_\lambda + \varepsilon_j - \varepsilon_{j'})\,,
$$

$$
\Delta\omega_{jj'} = \omega + iI + \varepsilon_j - \varepsilon_{j'}\,,
$$

$$
\Delta\varepsilon_{jj'} = \omega_\lambda + \varepsilon_j - \varepsilon_{j'}\,, \qquad (10)
$$

where I is an averaging parameter [12]). Thus our complete expression for the response to second order in the fermion-phonon coupling has 24 terms, 16 for the propagator correction, fig. 3, and 8 for the vertex correction, fig. 4.

Our interest is in the damping of the collective states due to mixing with the particle-hole plus phonon states [11,12]). These processes are described by the imaginary part of $P^{(2)}_{\rm ret}$ at the poles, where the real part of the energy $\Delta\omega_{\lambda 20}$ vanishes.

The first two terms in eq. (8a) have such a pole, and so they will be of particular interst to us. The vanishing energy denominator will only correspond to a physical situation if the single-particle state 2 is above the Fermi surface. In the zero-temperature limit then $n_{\rm B}(\lambda) = n_{\rm F}(2) = 0$ and $n_{\rm F}(0) = 1$, reducing eq. (8a) to the form

$$(P^{(2)}_{\rm ret}({\rm GR}, \omega + iI)_{\rm p} = \sum_{1,0} V^2(1, 0; {\rm GR}) \sum_{12,\lambda} V^2(1, 2; \lambda) \times \left\{ \frac{1}{\Delta\omega_{\lambda 20}} \left[\frac{1 - n_{\rm F}(1)}{(\Delta\omega_{01})^2} + \frac{n_{\rm F}(1)}{(\Delta\varepsilon_{21})^2} \right] \right\}. \tag{11}$$

They describe two different situations, depending on whether particle 1 is above or below the Fermi surface. If it is above, $n_{\rm F}(1) = 0$ and only the first term in eq. (11) survives. This is just the usual graph for fig. 3a in the indicated time ordering. When particle 1 is below the Fermi surface, the second term survives, which an energy denominator corresponding to the ground state correlation graph of fig. 5.

The last term in eq. (8a) may be easily interpreted from the expression for the RPA response, fig. 6,

$$(P^{(1)}_{\rm ret}({\rm GR}, \omega + iI))_{\rm p} = \sum_{1,0} V^2(1, 0; {\rm GR}) \frac{n_{\rm F}(1) - n_{\rm F}(0)}{\Delta\omega_{01}} \tag{12}$$

and for the particle self-energy, fig. 2a,

$$\Sigma^{(1)}_{\rm ret}(\varepsilon_1) = -\sum_{2,\lambda} V^2(1, 2; \lambda) \frac{1 + n_{\rm B}(\lambda) - n_{\rm F}(2)}{\Delta\varepsilon_{21}}. \tag{13}$$

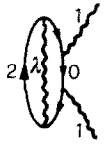

Fig. 5. A time-ordered $T = 0$ process associated with the diagram in fig. 3a at $T \neq 0$.

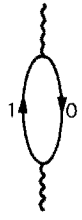

Fig. 6. Graphical represenation of the RPA Green function.

The interaction renormalizes the single-particle states, shifting the energy from ε_1 to $\varepsilon_1+\Sigma^{(1)}_{\rm ret}(\varepsilon_1)$. Then the argument of the occupation factor should also be shifted, which in the limit of a small perturbation gives an additive correction proportional to the derivative of the occupation factor,

$$\begin{aligned}(\overline{P^{(1)}_{\rm ret}}(\mathrm{GR},\omega+iI))_{\rm p} &= \sum_{1,0} V^2(1,0;\mathrm{GR})\frac{n_{\rm F}(\varepsilon_1+\Sigma^{(1)}_{\rm ret}(\varepsilon_1))-n_{\rm F}(0)}{\Delta\omega_{01}} \\ &\sim (P^{(1)}_{\rm ret}(\mathrm{GR},\omega+iI))_{\rm p}+\sum_{1,0} V^2(1,0;\mathrm{GR})\sum_{2,\lambda} V^2(1,2;\lambda) \\ &\quad\times\frac{(n_{\rm B}(\lambda)+1-n_{\rm F}(2))n'_{\rm F}(1)}{\Delta\omega_{01}\Delta\varepsilon_{21}}. \end{aligned} \tag{14}$$

The renormalization of the second occupation factor in eq. (12) is found in the similar terms in the hole propagator correction to the response, fig. 3b and eq. (8b). The non-trivial zero-temperature limit of terms of this kind, which involves derivatives of the fermion occupation factors are discussed in ref. [15]). To summarize, the zero-temperature limit of eq. (8a) has the expected form. However, one could not reconstruct the finite temperature expression by inserting the corresponding occupation factors in the zero-temperature expression as claimed in refs. [16-17]).

We next examine the zero-temperature limit of the vertex renormalization contribution, eq. (9). Placing particles 1 and 2 above the Fermi level and particles 0 and 3 below, we find after some simplification that eq. (9) reduces to

$$\begin{aligned}\lim_{T\to 0}(P^{(2)}_{\rm ref}(\mathrm{GR},\omega+iI)_v &= -\sum_{1,0} V(1,0;\mathrm{GR})\sum_{23} V(2,4;\mathrm{GR})\sum_{\lambda} V(12,30;\lambda) \\ &\quad\times\left\{\frac{1}{\Delta\omega_{01}\Delta\omega_{32}}\left(\frac{1}{\Delta\omega_{\lambda 20}}+\frac{1}{\Delta\omega_{\lambda 13}}\right)\right\}. \end{aligned} \tag{15}$$

The two terms are just the expressions obtained from the two time-ordered graphs shown in fig. 7. For our application to the damping of the giant vibrations, these together with graphs 3a and 3b, are the most important graphs. Other time orderings will only be important if the vibrational states are strongly collective in the sense that the backwards going amplitudes of the RPA solution are significant.

One unpleasant feature of our second order response is the presence of singularities from the energy denominators $\Delta\varepsilon_{jj'}$. The frequency ω does not appear in these energies and the singularity has no physical significance. In fact, the complete

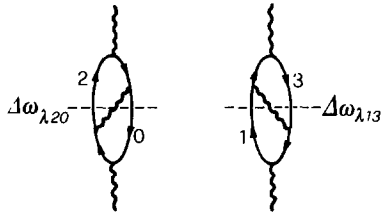

Fig. 7. The two time-ordered $T=0$ diagrams associated with the diagram shown in fig. 4 at $T\neq 0$.

expressions of eqs. (8) and (9) are well-behaved when these energies approach zero. The apparent singularity arises from the divergence of perturbation theory for degenerate levels. At zero temperature one never perturbs an occupied level into an unoccupied one at the same energy. However, this situation can arise at finite temperature.

We anticipate that the main effect of finite temperature on the response will be through the boson occupation factors. It is true that the fermion occupation factors are also varying, but the specifics of occupation probabilities is unimportant for the giant vibrations at zero temperature. To be important at finite temperature, it must act to increase the Landau damping, which is seen not to occur for temperatures below the shell gap energy [8]). A reasonable approximation to the finite temperature damping might be obtained by keeping only the temperature dependence of the boson occupation factors, and keeping only the poles associated with the vanishing of $\Delta\omega_{\pm\lambda ki}$, where k identifies a level above the Fermi surface and i one below it. The reduced expressions for (8a) and (9) are then

$$(P^{(2)}_{\rm ret}(\mathrm{GR},\omega+iI)_{\rm p}=\sum_{1,0}\frac{V^2(1,0;\mathrm{GR})}{(\Delta\omega_{01})^2}\sum_{12,\lambda}V^2(1,2;\lambda)\left(\frac{1+n_{\rm B}(\lambda)}{\Delta\omega_{\lambda 20}}+\frac{n_{\rm B}(\lambda)}{\Delta\omega_{-\lambda 20}}\right), \tag{16}$$

$$(P^{(2)}_{\rm ret}(\mathrm{GR},\omega+iI))_{\rm v}=-\sum_{12,34,\lambda}\frac{V(1,0;\mathrm{GR})V(12,30;\lambda)V(2,3;\mathrm{GR})}{\Delta\omega_{01}\Delta\omega_{32}}\times\left\{\left[\frac{n_{\rm B}(\lambda)+1}{\Delta\omega_{\lambda 20}}+\frac{n_{\rm B}(\lambda)+1}{\Delta\omega_{\lambda 13}}\right]+\left[\frac{n_{\rm B}(\lambda)}{\Delta\omega_{-\lambda 20}}+\frac{n_{\rm B}(\lambda)}{\Delta\omega_{-\lambda 13}}\right]\right\}. \tag{17}$$

This result is just what one would expect. The process in which a phonon is created in the intermediate state is enhanced by a factor $(1+n_{\rm B}(\lambda))$. In addition, a physical state may be created by annihilating a phonon already present. This appears in terms proportional to $n_{\rm B}(\lambda)$ with poles corresponding to $\Delta\omega_{-\lambda ki}=0$.

It is noted the similarity between the expressions (16) and (17) and that obtained from the functional differentiation of the fermion-phonon collision integral

$$I\sim\sum_{jj'\lambda}V^2(\lambda;jj')\{\delta(\varepsilon_{j'}-\omega_\lambda-\varepsilon_j)[n_{\rm F}(j')-n_{\rm F}(j)](1+n_{\rm B}(\lambda))+\delta(\varepsilon_{j'}+\omega_\lambda-\varepsilon_j)[n_{\rm F}(j')-n_{\rm F}(j)]n_{\rm B}(\lambda)\}, \tag{18}$$

as was done for the zero-temperature case in ref. [12]) [cf. also refs. [18,19])].

3. Numerical calculations

The properties of a nuclear excitation can be conveniently described in terms of its strength function. In the doorway approximation discussed in the previous section it can be written in the form [12])

$$S_a(E)=\frac{S_0}{\pi}\frac{\frac{1}{2}\Gamma(E+iI)+I}{[E_a-E-\Delta E_a(E+iI)]^2+[\Gamma(E+iI)/2+I]^2}, \tag{19}$$

where

$$\Delta E_a(E+iI)=\begin{cases}\mathrm{Re}\,(\Sigma^{(1)}_{\mathrm{ret}}(E+iI)) & \text{single-particles}\\ \mathrm{Re}\,(P^{(2)}_{\mathrm{ret}}(E+iI)) & \text{vibrations}\,,\end{cases} \tag{20}$$

$$\Gamma(E+iI)=\begin{cases}2\times\mathrm{Im}\,(\Sigma^{(1)}_{\mathrm{ret}}(E+iI)) & \text{single-particles}\\ 2\times\mathrm{Im}\,(P^{(2)}_{\mathrm{ret}}(E+iI)) & \text{vibrations}\,.\end{cases} \tag{21}$$

are the real and imaginary part of the self-energy of the nuclear excitation.

In the collective model the coupling between a vibration of multipolarity λ and a nucleon j' (cf. eq. (1)) is written as

$$V(j,j';\lambda)\equiv\langle(j';\lambda)j|V|j\rangle=\frac{1}{\sqrt{(2j+1)(2\lambda+1)}}\beta_\lambda\langle j\|R_0(\partial U/\partial r)\,Y_\lambda\|j'\rangle\,. \tag{22}$$

The quantities U and β_λ are the shell model potential and the deformation parameters associated with the vibration. These quantities, as well as the vibrational energies ω_λ were calculated by solving the RPA equation

$$1+kfG^0_\lambda(\omega,T)=0\,, \tag{23}$$

with

$$G^0_\lambda(\omega,T)=\sum_{ki}\frac{\langle k\|R_0(\partial U/\partial r)\,Y_\lambda\|i\rangle^2}{f^2}\times\frac{[n_{\mathrm{F}}(i)-n_{\mathrm{F}}(k)]}{2\lambda+1}([\varepsilon_k+\varepsilon_i-\omega]^{-1}+[\varepsilon_k-\varepsilon_i+\omega]^{-1})\,, \tag{24}$$

where

$$f=\int R_0\frac{\partial U}{\partial r}R_0\frac{\partial\rho}{\partial r}r^2\,\mathrm{d}r\,. \tag{25}$$

The sum in (24) is over particles k and holes i. Single-particle wavefunctions and energies from Hartree-Fock calculations with a Skyrme III interaction were utilized. The particle-hole space is chosen so as to satisfy the energy-weighted sum rule for a given multipolarity. The value of k is chosen to bring the isoscalar dipole state at zero energy and is of the order of unity. In the calculations of the self-energies reported below, all the RPA roots with multipolarity $\lambda=2,3,4$ and 5 and with an excitation energy $\leq$25 MeV were allowed in the intermediate states. For more details cf. ref. [11]).

We have used the computed vibrational properties to calculate $\mathrm{Im}\,(\Sigma^{(1)}_{\mathrm{ret}}(\varepsilon+iI))$ (cf. eq. (2)), for the valence neutron particle and hole states of ^{208}Pb at the Fermi energy, and obtain the curve shown in fig. 8. As anticipated, the temperature

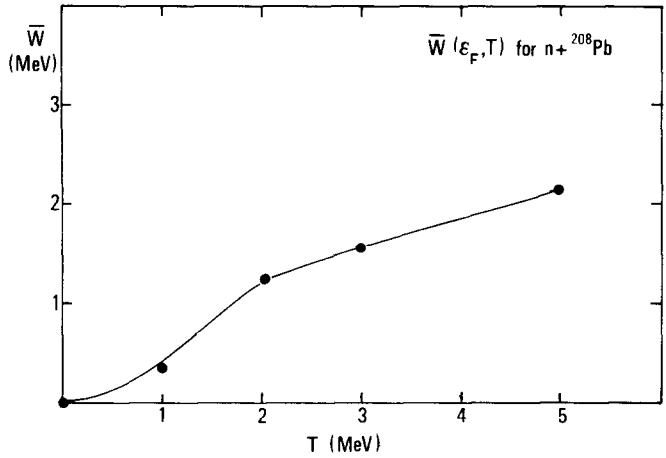

Fig. 8. The imaginary part $W = \mathrm{Im}\,(\Sigma^{(1)}_{\mathrm{ret}}(T, \varepsilon + iI))$ of the self-energy of neutron single-particle states in ^{208}Pb for $\varepsilon = \varepsilon_{\mathrm{F}}$. The quantity W was averaged over the valence orbitals j.

dependence of Γ is not quadratic. The obtained behavior is controlled by the boson occupation factor, as the quantities $\sum \beta^2_\lambda(n)$ are rather constant with temperature (cf. eq. (22) and table 1).

As already found at $T = 0$ [cf. ref. [11])] the largest contributions to Γ arise from the coupling to octupole vibrations. This is especially true at $T \neq 0$ because of the presence of an octupole vibration at essentially zero energy with a particularly large boson factor. In this case, the associated sum $\sum_n \beta^2_3(n)$ for vibrations with energies $\lesssim 7$ MeV changes by less than 25% in the energy range studied.

TABLE 1

Properties of the low-lying octupole vibrations of ^{208}Pb at $T = 3$ MeV

n	ω_n (MeV)	$\beta(n)$	$n_{\mathrm{B}}(n)$
1	0.60	0.013	4.5
2	0.80	0.04	2.0
3	0.94	0.04	1.7
4	3.90	0.06	0.4
5	4.10	0.06	0.34

In the second column the energy of the modes are shown while the associated deformation parameter and boson factor are displayed in columns three and four. Note that $(\Sigma_n \beta^2_3(n))^{1/2} \approx 0.10$ which is almost identical to the β_3-value associated with the single low-lying octupole vibration found at 2.6 MeV for $T = 0$.

Corrections due to Pauli principle violations, and implying boson occupation factors at the unperturbed particle-hole excitation energies [cf. refs. [10,11])], have been evaluated and amount to less than 10%.

A simple parameterization of the results at $T \neq 0$ and $\omega = \varepsilon - \varepsilon_F \neq 0$ is provided by

$$\Gamma(T, \omega) = a\omega^{\alpha} + bT^{\alpha} \tag{26}$$

with $\alpha \approx 1$, $a \approx 0.4$ and $b \approx 0.69$ for values of T and ω of a few MeV. This very approximated expression neglects the strong shell structure displayed by the results (cf., e.g., fig. 21 of ref. [12])). The linear dependence on ω and T is understandable following the arguments of the appendix, because the collective response function, summed over many multipolarities, is rather constant with energy.

The properties of the single-particle self-energy at finite temperature in the Brueckner-Hartree-Fock and in the Thomas-Fermi approximation have been recently discussed in ref. [20]).

In figs. 9 and 10 the strength function associated with the GDR of ^{90}Zr and the GQR of ^{208}Pb are shown for two temperatures. Even at $T = 3$ MeV the position of the peaks hardly moves and their widths have become, if anything, smaller. In both cases however, the area under the peak has been reduced by about 30%. This strength is now found at lower excitation energies, over an energy range of several MeV. This is because at $T \neq 0$, both $(P^{(2)}_{\mathrm{ret}}(\omega + iI))_{\mathrm{p,h}}$ and $P^{(2)}_{\mathrm{ret}}(\omega + iI))_{\mathrm{v}}$ (cf. eqs. (8) and (9)) display poles at $\omega = (\varepsilon_{\mathrm{p}} - \varepsilon_{\mathrm{h}}) + \omega_{\lambda}$ and $\omega = (\varepsilon_{\mathrm{p}} - \varepsilon_{\mathrm{h}}) - \omega_{\lambda}$. While the first energy coincides with the energy of the peak of the giant resonance at $T = 0$, the second value of ω is shifted down from this value by $2\omega_{\lambda}$. This result implies that the damping of the full strength function has increased.

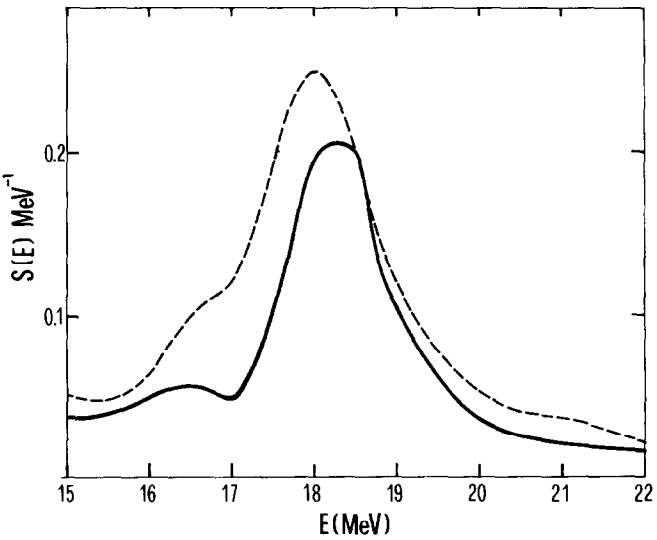

Fig. 9. Strength function in the interval region of the main peak of the GDR of ^{90}Zr at $T = 0$ (dashed line) and $T = 3$ MeV (full line).

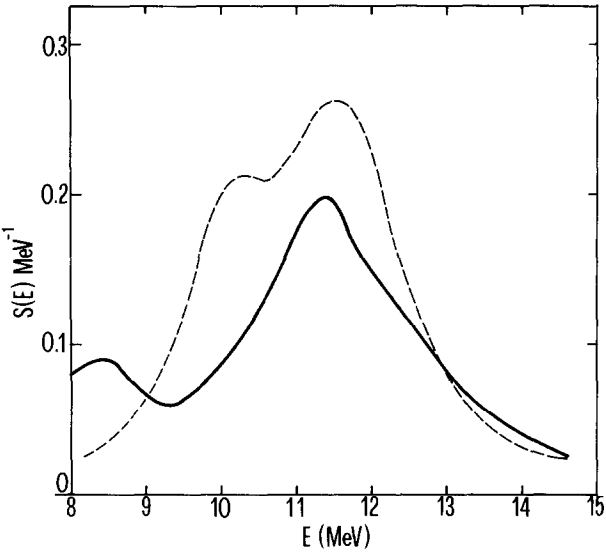

Fig. 10. Strength function for the isoscalar giant quadrupole resonance of ^{208}Pb at $T = 0$ (dashed line) and $T = 3$ MeV (full line).

It is an open question whether the low-energy tail can account for the missing strength reported experimentally [4,5]).

The fact that the temperature dependence of the spreading width is weaker than quadratic is at variance with the result of ref. [9]), in the case of an infinite system. It is however consistent with the results discussed above for the single-particle damping and reflects the central role the nuclear surface plays in the spreading mechanism [12,14]).

4. Conclusion

Giant resonances seem to be extremely resilient to temperature. One can thus conclude that the sizeable changes observed in the damping width of these vibrations arise mainly from changes in the deformation of the average nuclear field induced under the stress of angular momentum and temperature [cf. ref. [21]) and references therein].

Appendix

In this appendix, we present a simple heuristic derivation of the temperature dependence of Im $(\Sigma^{(1)}(T, \omega))_p$ for the case of infinite system (cf. eq. (5) and fig. 11).

At the Fermi surface, we may consider the temperature ($T \ll \varepsilon_F$) as the energy available for real transitions to the 2p-1h intermediate states (cf. fig. 11). These transitions are allowed, because the Fermi surface has a diffuseness of the order of

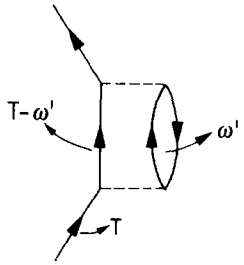

Fig. 11. Pictorial representation of the lowest order process contributing to $\mathrm{Im}\,(\Sigma^{(1)}_{\mathrm{ret}}(T,\omega))_{\mathrm{p}}$.

T. We thus do not need to explicitly introduce the fermion occupation factors. Using the fact that the uncorrelated particle-hole response function depends linearly on the energy ω' in the considered energy range [22]), one can write

$$\mathrm{Im}\,(\Sigma^{(1)}(T,\omega=0))_{\mathrm{p}} \sim \int_0^T \omega'\,\mathrm{d}\omega' = \tfrac{1}{2}T^2\,, \tag{A1}$$

thus leading to the quadratic dependence displayed by eq. (5).

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