Implications of the recently discovered ^{208}Pb(1^+,$E_x = 5.845$ MeV) state for the spin-dependent nuclear interaction

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Measurements on the newly discovered 1^+ state at $E_x = 5.845$ MeV in ^{208}Pb provide an opportunity to examine the spin dependence in the nuclear effective Hamiltonian. The data require a somewhat weaker spin-orbit splitting than given by conventional Hamiltonians. The isoscalar spin-dependent residual interaction, which was poorly determined by previous data, is found to be much weaker than the isovector spin-dependent interaction, as predicted by Brueckner theory.

[NUCLEAR STRUCTURE Simple shell model, spin parameters in the Hamiltonian for ^{208}Pb.]

Very recently, groups at Osaka[1] and at Giessen[2] independently identified a long-sought low-lying 1^+ state in ^{208}Pb, which we will call the isoscalar *M*1 state. Hayakawa *et al.*[1] made a search for 1^+ states between 4 and 7 MeV excitation using the (p, p′) reaction. Only one state was found in this region, at an excitation energy of 5.845 MeV. The spectroscopic factor for the reaction ^{209}Bi(d, ^{3}He)^{208}Pb was then measured to determine the proton $h_{9/2}$-$h_{11/2}^{-1}$ component of the wave function, and the amplitude was measured to be larger than 0.87. Weinhard *et al.*[2] studied ^{208}Pb excitations using the nuclear resonance fluorescence technique with linearly polarized photons. They assigned a 1^+ state at $E_x = 5.845$ MeV and further reported the transition strength $B(M1)\uparrow = (1.6 \pm 0.5)\ \mu_N^2$, assuming that the 1^+ state has no other decay modes than the *M*1 transition to the ground state.

This state is predicted in simple shell model theory; its significance is due to its structure as the isoscalar combination of neutron and proton spin-flip configurations,

$$|1_1^+\rangle = a_\pi|\pi h_{9/2}\pi h_{11/2}^{-1}\rangle + (1-a_\pi^2)^{1/2}|\nu i_{11/2}\nu i_{13/2}^{-1}\rangle \ , \quad (1)$$

with a_π positive. The properties of the state are therefore sensitive to the poorly known isoscalar spin-dependent interaction. There have been many theoretical studies of 1^+ states in ^{208}Pb,[3–5] predicting the properties of the state based on various Hamiltonian models. We want to turn this process around as far as possible and use the data to infer the interaction. For reference, we quote the theoretical results of Vergados.[3] He predicted a proton amplitude of $a_\pi = 0.78$ in the wave function, an excitation energy of $E_x = 5.45$ MeV, and a transition rate $B(M1)\uparrow = 1.2\ \mu_N^2$. This rate agrees with experiment, but the calculation did not take into account the quenching of the spin moments due to configurations outside of the shell model space. Also, the proton amplitude predicted by Vergados appears to be too small.

We will now argue that the resonance fluorescence experiment is consistent with the pickup experiment, giving a proton amplitude in the range $a_\pi \sim 0.87$–0.9, once the quenching is taken into account. To simplify our discussion, we will consider only the two configurations in Eq. (1). This simplification is supported by the calculation of Ref. 3, which indicates that the admixture of higher particle-hole states is only 2%, and by our own calculations, which find that the higher particle-hole states result in an energy shift of the lowest state by only a few keV. Also, the random-phase approximation correlations are quite small. However, Wambach, Jackson, and Speth[6] have pointed out that there might be strong coupling to higher shells via tensor forces, which are not included in our calculations. In any case, the effective interaction parameters we obtain are those defined in a small model space. To relate the $B(M1)$ value to the proton amplitude, we note the *M*1 transition matrix elements

$$\mu_\pi = \langle\pi||0(M1)||0\rangle = 3.02\ \mu_N \ ,$$
$$\mu_\nu = \langle\nu||0(M1)||0\rangle = -2.74\ \mu_N \ , \quad (2)$$

We have used free nucleon magnetic moments in the above. The $B(M1)$ value can then be expressed in terms of the proton amplitude a_π as

$$B(M1)\uparrow = 3|a_\pi\mu_\pi + (1-a_\pi^2)^{1/2}\mu_\nu|^2 \ . \quad (3)$$

The relation defined by Eq. (3) between the $B(M1)$ and a_π is shown in Fig. 1. The experimental $B(M1)$ value appears to demand a proton amplitude in the range $a_\pi = 0.79 \pm 0.015$. However, the spin matrix elements for this state should be quenched to the same extent as for the better known isovector *M*1 excitations. Systematic studies of magnetic[7] as well as Gamow-Teller transitions[8] demand a quenching factor of about $\frac{1}{3}$ in the medium and heavy nuclei. This quenching phenomenon was predicted many years ago,[9,10] and recent calculations[11,12] are consistent with experimental findings. Thus we renormalize the experimental $B(M1)$ value by a factor of 3 and obtain $a_\pi = 0.84$–0.9 as indicated in Fig. 1. This value is quite consistent with the pickup measurement, and so we shall demand that the Hamiltonian reproduce both bounds. [A similar conclusion is reached if we renormalize the spin moments or the isovector part of the spin moments in Eq. (2) by a factor of $\sqrt{3}$.]

The Hamiltonian matrix in the small space (1) has four elements, and we have so far two pieces of data, the energy of one of the eigenstates and its wave function. Two more pieces of information are needed to determine all the quantities; we shall use our knowledge of the isovector interac-

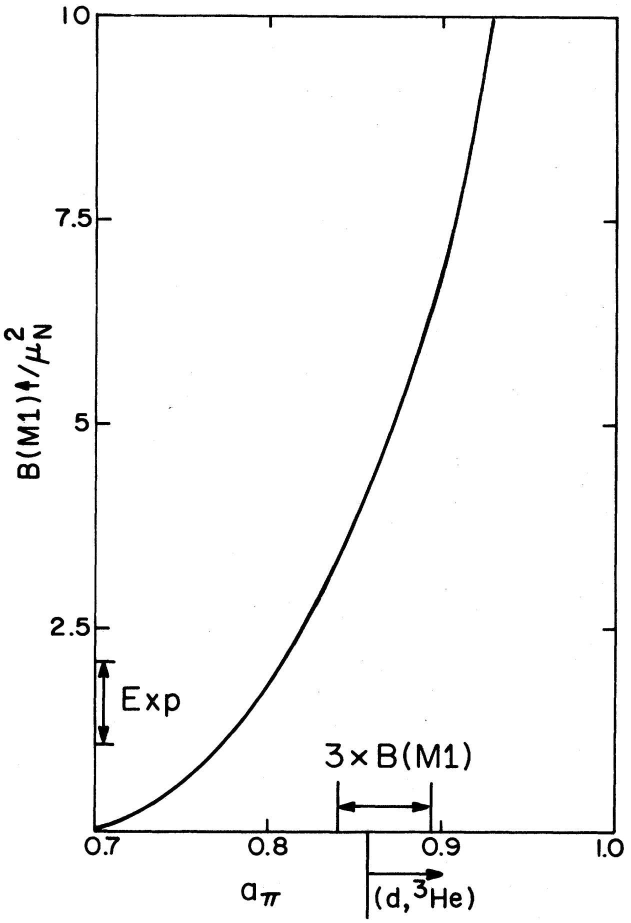

FIG. 1. The dependence of the $B(M1)$ in the lower 1^+ state of ^{208}Pb as a function of the proton amplitude a_π, according to Eq. (2). The lower bound for a_π obtained by the (d,^{3}He) reaction (Ref. 1) is shown on the horizontal axis. The range of a_π corresponding to three times the experimental $B(M1)$ is also shown.

tion and corresponding eigenstate. Following the Landau-Migdal formalism,[13] the interaction is conveniently parametrized as the direct matrix element of a delta function:

$$V_{ph}=V_\sigma\delta(\vec{r}_1-\vec{r}_2)\vec{\sigma}_1\cdot\vec{\sigma}_2+V_{\sigma\tau}\delta(\vec{r}_1-\vec{r}_2)\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2 \ . \tag{4}$$

The systematics of the isovector spin-dependent interaction $V_{\sigma\tau}$ are now well known from studies of the giant Gamow-Teller states. The study of Bertsch, Cha, and Toki[14] shows that $V_{\sigma\tau}$ is in the range of $V_{\sigma\tau}=200$–220 MeV fm^3. We shall calculate the isovector interaction using the range of values for the strength. In the calculation, we use the single-particle wave functions of the Skyrme III Hartree-Fock Hamiltonian.[15] In fact, the results are not sensitive to the choice of single-particle Hamiltonian, with Woods-Saxon eigenstates and Skyrme IV Hartree-Fock orbitals giving matrix elements within 5%, as may be seen from Table I. With the isovector matrix elements and the data on the 5.85 MeV state, there is a close constraint on the isoscalar effective interaction and the proton spin-orbit splitting. This is

TABLE I. The particle-hole matrix elements of a delta-function interaction in the active particle-hole configuration of ^{208}Pb. Matrix elements are quoted in units of 10^{-3} fm^{-3},

$$\langle ph|\delta(\vec{r}_1-\vec{r}_2)\vec{\sigma}_1\cdot\vec{\sigma}_2|p'h'\rangle \ .$$

Single particle model	π-π	ν-ν	π-ν
Woods-Saxon	5.06	5.30	5.12
Skyrme III	5.06	5.61	5.30
Skyrme IV	4.90	5.22	4.97

obvious because the state has a predominant proton character as well as isoscalar character. The relationship is shown in Fig. 2. We see that the data are consistent with $V_\sigma \sim 0$ if the proton spin-orbit splitting is about 5.3 MeV. Alternatively, the data permit $V_\sigma \sim V_{\sigma\tau}$ if the proton spin-orbit splitting is less than 4 MeV, which would be extremely weak.

We shall now invoke the empirical information on the rest of the $M1$ strength to place sharper bounds on the interactions. Of the total strength permitted in the model space of Eq. (2), $\frac{1}{5}$ has been found,[16] and is located at an excitation energy of 7.5 MeV. We would like to assign this to the upper state in the 2×2 Hamiltonian, but some

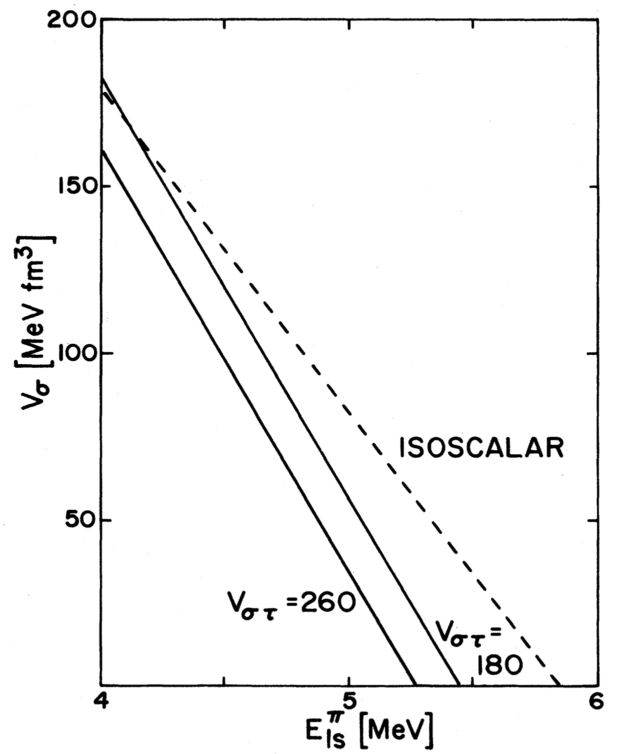

FIG. 2. The relation between the isoscalar interaction strength and the proton spin-orbit energy required for $E_1=5.85$ MeV. The two solid lines show the relation, given the proton amplitude $a_\pi=0.85$. In the isoscalar limit ($a_\pi=\sqrt{0.5}$), shown as the dashed line, the lower state is independent of the isovector interaction.

TABLE II. Spin-dependent interaction inferred from the properties of the spin excitations in ^{208}Pb. The assumed parameters are $E_1=5.845$ MeV, $a_\pi=0.87$–0.9, $V_{\sigma\tau}=200$–220 MeV fm^3, $E_2=7.7$–8.2 MeV.

	This work	Other determinations		
V_σ	30–100 MeV fm^3 $\sim\frac{1}{7}-\frac{1}{2}V_{\sigma\tau}$	$\sim\frac{1}{3}V_{\sigma\tau}$[a] Skyrme III	$\sim V_{\sigma\tau}$[b] Skyrme IV	Optical model
E_{ls}^π	4.6 –5.1 MeV	5.4	7.1	5.6
E_{ls}^ν	5.4 –6.2 MeV	7.4	9.5	6.2
E_{ls}^ν/E_{ls}^π	1.16–1.26	1.37	1.34	1.10

[a]Reference 16. [b]Reference 17.

strength is probably missing since, according to the empirical quenching, we expect to find $\frac{1}{3}$ of the shell model limit in the upper state. We may conservatively assume that the missing strength lies in the region of 8.0–9.0 MeV, in which case the mean energy of the upper configuration is in the range 7.7–8.2 MeV. Using these values for the second eigenenergy, we deduce the spin-dependent interaction, with results summarized in Table II. We find that the isoscalar interaction strength would be in the range $V_\sigma \sim \frac{1}{7}-\frac{1}{2}V_{\sigma\tau}$. This is quite consistent with the reaction matrix interaction deduced from the Reid soft-core potential, which has the prediction $V_\sigma \sim \frac{1}{3}V_{\sigma\tau}$.[17] The more simplified model of interaction based on pi and rho exchange predicts $V_\sigma \sim V_{\sigma\tau}$,[18] which is inconsistent with our analysis. We can infer from Vergados's calculation that his V_σ was close to zero. Other phenomenological analyses have produced larger values of V_σ. A global fit[4] to magnetic properties of many low-lying states was obtained with $V_\sigma \sim V_{\sigma\tau}$. Our analysis of the $M1$ excitation in ^{90}Zr determined a spin-dependent interaction for neutron excitations that was only slightly weaker than for isovector excitations.[19] However, because of the sensitivity to the spin-orbit interaction, the new data provide more reliable bounds.

The analysis permits us to infer the spin-orbit splittings of proton and neutron orbits separately. The results are given in Table II and compared with values obtained by other methods. One striking thing to notice is that the proton spin-orbit splitting is significantly smaller than the neutron splitting. The difference in orbital angular momenta produces an effect in this direction:

$$\frac{l_\nu - l_\pi}{l_\pi} = 20\% \ .$$

However, in the phenomenological potential of Becchetti and Greenless,[20] the radial overlaps go in the opposite direction and the spin-orbit splittings are predicted to be more nearly equal. From Table II it may be seen that the Skyrme Hamiltonians predict a difference between neutrons and protons larger than obtained in our phenomenological analysis. Much of the neutron-proton difference in the Skyrme Hamiltonian is due to the assumed P-wave origin of the two-body spin-orbit interaction.[21] The numbers of P-wave pairs for protons and neutrons differ in heavy nuclei, and the ratio of interaction strengths depends on N and Z as

$$\frac{V_{ls}^\nu}{V_{ls}^\pi} = \frac{2N+Z}{2Z+N} \ . \quad (5)$$

The optical-model based phenomenological potential does not include this specific isospin dependence, which we believe ought to be present to some extent. It would be useful to reexamine the optical potentials, allowing a possible isospin dependence to the spin-orbit potential, to further study this question.

In conclusion, we argue that the proton amplitude of the lowest 1^+ state in ^{208}Pb has a magnitude $a_\pi \sim 0.87$–0.9 based on two independent experiments. This, together with the energy of the state and previously known data on the isovector spin excitations, determines some interesting bounds on the isoscalar interaction strength V_σ and on the spin-orbit splittings. The isoscalar interaction appears to be small but nonzero, consistent with Brueckner theory. The isospin dependence of the spin-orbit splitting is sufficiently determined to warrant renewed microscopic study of the spin-orbit potential.

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