

## AN EFFECTIVE INTERACTION FOR INELASTIC SCATTERING DERIVED FROM THE PARIS POTENTIAL

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**Abstract:** An effective interaction for inelastic scattering of nucleons from nuclei is derived by fitting oscillator  $G$ -matrix elements of the Paris nucleon-nucleon potential to the matrix elements of a sum of Yukawa terms. Except for the singlet-odd channel, these  $G$ -matrix elements do not differ in any significant respect from those obtained from the Reid soft-core potential, and give similar results for inelastic proton scattering.

### 1. Introduction

Some years ago, effective interactions for inelastic scattering of nucleons from nuclei were derived by Bertsch *et al.*<sup>1)</sup> from the phenomenological nucleon-nucleon (NN) potentials of Hamada and Johnston<sup>2)</sup> and Reid<sup>3)</sup>. The derivation proceeded in two stages: the  $G$ -matrix elements of these potentials in an oscillator basis were first obtained, and then fitted to a sum of Yukawa terms. The effective interactions thus calculated were appropriate for use in nucleon scattering codes and have been applied to several inelastic nucleon scattering studies at bombarding energies up to about 65 MeV [refs. 4–6)]. A particular choice of the interaction which has come to be known<sup>4)</sup> as M3Y has been especially popular.

The purpose of the present paper is to describe a similarly motivated effective interaction derived from the Paris NN potential<sup>7)</sup>, an analytic form for which has recently been published<sup>8)</sup>. The medium- and long-range parts of this potential are based upon a meson theory of nuclear forces, while the short-range part is fitted phenomenologically. The Paris potential is thus based on a more fundamental theory of the NN interaction than the earlier potentials. Moreover, new NN scattering data obtained in the 1970's went into the development of this potential. The Reid soft-core potential is based on earlier and partially erroneous phase-shift data in the singlet-odd, triplet-even and spin-orbit-even channels<sup>9)</sup>; and it has a much stronger tensor part than the Paris potential<sup>10)</sup>. It is of interest to study whether these differences in the basic NN potential lead to any significant changes in the effective interaction.

The  $G$ -matrix elements that we obtain in the course of the derivation of the

effective interaction are interesting in their own right and can be used in different applications, such as nuclear structure calculations and the extraction of the parameters of the Landau-Migdal force. In this paper, we focus on the application to inelastic nucleon scattering.

## 2. Theoretical procedure

### 2.1. G-MATRIX ELEMENTS IN THE HARMONIC OSCILLATOR BASIS

From the Paris NN potential,  $G$ -matrix elements in the harmonic oscillator basis were obtained by the method of Barrett, Hewitt and McCarthy (BHM)<sup>11</sup>). In order to clarify the method and the approximations used, the formalism developed by these authors is briefly outlined here. We start with the two-body hamiltonian

$$H = H_0 + V, \quad (1)$$

where the zeroth-order hamiltonian  $H_0$  provides the harmonic oscillator wave functions and energies by solution of the Schrödinger equation,

$$H_0 \phi_\mu = \varepsilon_\mu \phi_\mu, \quad (2)$$

for the two-body system. The nucleon-nucleon interaction  $V$ , which may be ill-behaved around the origin of the relative coordinate, is then treated by the Bethe-Goldstone (BG) equation<sup>12</sup>),

$$|\Psi_\alpha^{\text{BG}}(\omega)\rangle = |\phi_\alpha\rangle + \sum_\mu |\phi_\mu\rangle \frac{Q_\mu}{\omega - \varepsilon_\mu} \langle \phi_\mu | V | \Psi_\alpha^{\text{BG}}(\omega) \rangle. \quad (3)$$

Here the Pauli exclusion operator  $Q_\mu$  ensures that neither of the two particles falls into the occupied states in the nuclear ground state. The reaction matrix  $G_{\mu\alpha}(\omega)$  is defined by the matrix elements of  $V$  between the unperturbed state  $\phi_\mu$  and the exact solution  $\Psi_\alpha^{\text{BG}}$ :

$$G_{\mu\alpha}(\omega) = \langle \phi_\mu | V | \Psi_\alpha^{\text{BG}}(\omega) \rangle. \quad (4)$$

The BG equation (3) is not very useful as it stands for constructing the  $G$ -matrix, for the procedure converges very slowly. BHM found it more convenient to introduce an expansion of  $\Psi_\alpha^{\text{BG}}(\omega)$  in terms of the complete set of eigenfunctions  $\psi_i$  of the Schrödinger equation

$$(H_0 + V)\psi_i = E_i\psi_i,$$

$$\Psi_\alpha^{\text{BG}}(\omega) = \sum_i^{\infty} a_{i\alpha}(\omega)\psi_i. \quad (5)$$

By this means they expected to get much better convergence than the harmonic oscillator expansion, because the functions  $\psi_i$  already contain the effect of the strong short-range repulsion part of  $V$ . Introducing further the overlaps

$$b_{i\mu} = \langle \psi_i | \phi_\mu \rangle, \quad (6)$$

they arrived at the following expression for  $G$  by a straightforward manipulation:

$$G_{\beta\alpha}(\omega) = G_{\beta\alpha}^R(\omega) - \sum_{\mu} G_{\beta\mu}^R(\omega) \frac{1 - Q_{\mu}}{\omega - \varepsilon_{\mu}} G_{\mu\alpha}(\omega). \quad (7)$$

The reference  $G$ -matrix in eq. (7) is defined by

$$G_{\beta\alpha}^R(\omega) = (\varepsilon_{\alpha} - \omega) \sum_i \frac{E_i - \varepsilon_{\beta}}{E_i - \omega} b_{i\alpha} b_{i\beta}, \quad (8)$$

which can be easily computed. Eqs. (7) and (8) correspond to the standard relation<sup>13)</sup> between the  $G$ -matrix and the reference matrix  $G^R$ .

The exact treatment of eqs. (5)–(8), and in particular the treatment of the Pauli operator, leads BHM to expressions which are complicated to deal with; see eqs. (28) and (29) of ref.<sup>11)</sup>. We prefer instead to use the approximation scheme introduced by Eden and Emery<sup>14)</sup>. By approximating the Pauli operator  $Q_{\mu}$ , they derive an integral equation for the  $G$ -matrix in only the relative coordinate, as we now discuss.

$Q_{\mu}$  in eqs. (3) and (7) is a product of single-particle Pauli operators  $Q(2n+l)$  which are given by

$$Q(2n+l) = \begin{cases} 0 & \text{for } 2n+l \leq N_{\max} \\ 1 & \text{for } 2n+l > N_{\max}. \end{cases} \quad (9)$$

Here  $N_{\max}$  is the principal quantum number of the highest occupied level. Eden and Emery replaced the exact Pauli operator  $Q_{\mu} = Q(N_1)Q(N_2)$  by

$$Q_{\mu} = Q(N_1 + N_2) = Q(N + \bar{N}) = \begin{cases} 0 & \text{for } N + \bar{N} \leq 2N_{\max} + 2 \\ 1 & \text{for } N + \bar{N} > 2N_{\max} + 2. \end{cases} \quad (10)$$

Here  $N = 2n+l$  and  $\bar{N} = 2\bar{n}+\bar{l}$  are the principal quantum numbers of the relative and center-of-mass wave functions, respectively. This prescription excludes the states wherein only one particle is excited from filled shells to the first or the second unfilled shell. However, the states with  $N_1 = N_{\max} + 1$ ,  $N_2 = N_{\max} + 1$ , which should be allowed, are improperly excluded. Furthermore, all states (except a few) involving excitation to higher shells are allowed, whereas in fact those states

for which one of the particles is in a filled shell should be excluded. The approximation seems to be a good one, but we shall return to it in the discussion of the numerical results. We note that this approximation is the analog of the angle-averaged Pauli approximation in nuclear matter theory<sup>12)</sup>.

Using the above approximation for the Pauli operator and noting that the center-of-mass wave functions are not affected by the two-body potential  $V$ , we arrive at the  $G$ -matrix equation in only the relative coordinate<sup>14)</sup>:

$$G_{\beta\alpha}(\omega) = G_{\beta\alpha}^R(\omega) - \sum_{\mu} G_{\beta\mu}^R(\omega) \frac{1 - Q(N_{\mu} + \bar{N}_{\alpha})}{\varepsilon - \varepsilon_{\mu}} G_{\mu\alpha}(\omega). \quad (11)$$

In eq. (11), as also in the earlier equations, all the suffixes are now to be considered to be those for the relative motion. The initial two-particle state is chosen to be one at the Fermi surface, since such states are the most important ones in nuclear structure calculations. Then  $\bar{N}_{\alpha}$  can be expressed in terms of the principal quantum numbers for relative motion using energy conservation:

$$\hbar\omega(N_1 + N_2) = \hbar\omega(N_{\alpha} + \bar{N}_{\alpha}) = 2N_{\max}\hbar\omega. \quad (12)$$

Hence  $\bar{N}_{\alpha} = 2N_{\max} - N_{\alpha}$ . Substitution of this into the definition of  $Q(N + \bar{N})$  gives

$$Q(N_{\mu}) = \begin{cases} 0 & \text{for } N_{\mu} \leq N_{\alpha} + 2 \\ 1 & \text{for } N_{\mu} > N_{\alpha} + 2. \end{cases} \quad (13)$$

Now we are in a position to discuss the Schrödinger equation for the relative wave function  $\psi_i$  in eq. (5). The explicit form is

$$[-\hbar^2\nabla^2/m + \frac{1}{4}m\omega^2r^2 + V(r)]\psi_i = E_i\psi_i, \quad (14)$$

where the oscillator constant  $\omega$  depends on the nuclear system considered. The potential  $V(r)$  in general contains central, spin-orbit, tensor and quadratic spin-orbit terms<sup>2,3,8)</sup>. The equation is solved for eigenenergy  $E_i$  and eigenfunction  $\psi_i$  in various two-nucleon channels ( $^1S_0$ ,  $^3P_1$ , etc.). For the  $^3S_1 - ^3D_1$  channels, which are coupled by the tensor term in  $V(r)$ , a pair of coupled differential equations was solved.  $\psi_i$  has a high degree of overlap with some oscillator function  $\phi_{nl}$ . The free  $G$ -matrix elements were obtained from the relation<sup>11,15)</sup>

$$G_{nl}^0 = \langle \phi_{nl} | G | \phi_{nl} \rangle = E - E_{nl}, \quad (15)$$

where  $E_{nl}$  is the oscillator energy in state  $\phi_{nl}$ . They were then corrected for the Pauli blocking effect as discussed above.

A special feature of the Paris potential is its momentum dependence. This could

be treated readily by using the relation

$$\langle x | \frac{d}{dx} | x' \rangle \simeq \frac{1}{\Delta} [\delta(x - x' + \frac{1}{2}\Delta) - \delta(x - x' - \frac{1}{2}\Delta)]. \quad (16)$$

This form led to the usual definition of the derivative,

$$\frac{d}{dx} \psi(x) \equiv \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} [\psi(x + \frac{1}{2}\Delta) - \psi(x - \frac{1}{2}\Delta)] \simeq \sum_{x'} \langle x | \frac{d}{dx} | x' \rangle \langle x' | \psi \rangle, \quad (17)$$

where  $\Delta$  is a small distance. In fact, good numerical accuracy was achieved only with a very small value of  $\Delta$  at small  $x$ :  $\Delta = 0.01$  fm for  $x \leq 1$  fm.

The  $G$ -matrix elements in the  $^1S_0$  and  $^1P_1$  channels gave respectively the singlet-even (SE) and singlet-odd (SO) components, while the coupled  $^3S_1 - ^3D_1$  channels gave the triplet-even (TE) and tensor-even (TNE) components. The triplet-odd (TO), tensor-odd (TNO), and the two components of the spin-orbit force were obtained from the following relations<sup>1)</sup>, which are derived by ignoring the quadratic spin-orbit term:

$$\begin{aligned} V(\text{TO}) &= V(^3P_0) + 2V(\text{LSO}) + 4V(\text{TNO}), \\ V(\text{TNO}) &= -\frac{5}{72} [2V(^3P_0) - 3V(^3P_1) + V(^3P_2)], \\ V(\text{LSO}) &= -\frac{1}{12} [2V(^3P_0) + 3V(^3P_1) - 5V(^3P_2)], \\ V(\text{LSE}) &= \frac{1}{3} [V(\text{TE}) - 2V(\text{TNE}) - V(^3D_1)]. \end{aligned} \quad (18)$$

## 2.2. FITTING OF EFFECTIVE INTERACTION TO $G$ -MATRIX ELEMENTS

The effective interaction was taken to be of the form

$$V = \begin{cases} \sum_i V_i Y(r_{12}/R_i), & \text{central} \\ \sum_i V_i Y(r_{12}/R_i) \mathbf{L} \cdot \mathbf{S}, & \text{spin-orbit} \\ \sum_i V_i r_{12}^2 Y(r_{12}/R_i) S_{12}, & \text{tensor,} \end{cases} \quad (19)$$

where  $i \leq 4$ ,  $Y(x) = e^{-x}/x$ , and the tensor operator  $S_{12}$  and the spin-orbit operator  $\mathbf{L} \cdot \mathbf{S}$  have the conventional normalization. This is the form conventionally used in microscopic distorted wave programs for inelastic scattering of nucleons. The choice of the ranges was theoretically motivated, as discussed in ref. <sup>1)</sup>, and satisfied the requirement that the central part of the interaction had the one-pion exchange potential tail. The strengths  $V_i$  were then determined by fitting to  $G$ -matrix elements in the various two-body channels by a least-squares fitting procedure.

### 3. Results

#### 3.1. G-MATRIX ELEMENTS

Because of the momentum dependence of the Paris potential, the computer programming required to obtain the free  $G$ -matrix elements was more complicated than for the potentials used in ref. <sup>1)</sup>. Some checks were therefore made on the programming. For the uncoupled channels, NN phase shifts were obtained from wave functions calculated in a very shallow oscillator potential ( $\hbar\omega = 0.001$  MeV) with a large value of 10 fm for the matching radius (beyond which the wave function was approximated by a plane wave). It was checked that these phase shifts agreed with the values listed in ref. <sup>8)</sup>. For the coupled  ${}^3S_1$ - ${}^3D_1$  calculation, it was verified that the deuteron binding energy and wave function were correctly reproduced.

The free  $G$ -matrix elements were calculated for an oscillator parameter  $\hbar\omega = 14$  MeV, which is appropriate for the region near  ${}^{16}\text{O}$ . The next step was to correct them for the Pauli blocking effect. An energy gap of 40 MeV between occupied and unoccupied levels was used. The resulting effective  $G$ -matrix elements are listed in table 1, labelled by the radial quantum numbers  $n$  and  $n'$  for the relative motion. As a check on the stability of these values, the effect of changing the energy gap to 30 MeV was investigated. This resulted in  $\sim 10\%$  change in the values quoted. The changes caused by replacing the Pauli blocking condition (10) by the requirement that  $Q_\mu$  be zero for  $(N + \bar{N}) \leq (N_{\text{max}} + 1)$  were also at the 10% level. An independent check on these numbers is obtained from the variational calculation of  $G$ -matrix elements of the Paris potential performed by Vary <sup>16)</sup>, which should give an upper bound for these values. His numbers are in fact very close to ours.

It is of interest to compare the effective  $G$ -matrix elements given in table 1 with the corresponding numbers from the Reid potential, which are given in ref. <sup>1)</sup> for the SE, TE, TNE and LSO channels. In these channels, the two sets of numbers agree with each other to within 10%, except for the case of the column headed  $n = 3$  for the SE channel. For the SO, TO, TNO and LSE channels, the matrix elements are smaller and are poorly determined by the NN interaction, which is why they are not given in ref. <sup>1)</sup>. But we have computed them for both the Paris and the Reid potential and again find that they are in agreement with each other to about 10%, except for the SO channel. This agreement in the two sets of numbers is worth emphasizing. It shows that the differences that exist between the Paris NN potential and the phenomenological Reid potential do not persist in the corresponding  $G$ -matrix elements, which are the quantities of interest in most applications. We note in particular that the matrix elements in the tensor-even and tensor-odd channels are very similar. Only for the SO channel and in the case of the column headed  $n = 3$  for the SE channel are there significant differences. But, since they occur mainly for highly nondiagonal matrix elements, these differences should not noticeably affect inelastic scattering results.

TABLE 1  
Oscillator  $G$ -matrix elements (in MeV) of the Paris potential ( $\hbar\omega = 14$  MeV)

	S	S	$n = 0$	1	2	3
singlet even	$n' = 0$		-6.73	-5.27	-3.73	-11.28
	1			-4.84	-3.47	-6.46
	2				-2.61	-1.44
	3					2.51
	S	S	$n = 0$	1	2	3
triplet even	$n' = 0$		-9.63	-8.90	-7.55	-6.07
	1			-8.43	-6.82	-5.63
	2				-5.83	-5.02
	3					-2.82
	P	P	$n = 0$	1	2	3
singlet odd	$n' = 0$		2.52	2.37	2.11	1.89
	1			3.03	3.00	2.81
	2				3.39	3.41
	3					3.74
	P	P	$n = 0$	1	2	3
triplet odd	$n' = 0$		-0.08	-0.23	-0.09	-0.05
	1			-0.24	-0.05	0.01
	2				0.04	0.13
	3					0.31
	S	D	$n = 0$	1	2	3
tensor even	$n' = 0$		-5.54	-7.62	-8.66	-9.21
	1		-2.61	-5.09	-6.90	-8.11
	2		-1.35	-2.95	-4.72	-6.23
	3		-0.72	-1.66	-2.98	-4.39
	P	P	$n = 0$	1	2	3
tensor odd	$n' = 0$		0.78	0.75	0.63	0.54
	1			0.92	0.87	0.79
	2				0.95	0.93
	3					1.00
	D	D	$n = 0$	1	2	3
LS even	$n' = 0$		-0.14	-0.13	-0.12	-0.21
	1			-0.18	-0.21	-0.47
	2				-0.26	-0.70
	3					-0.91
	P	P	$n = 0$	1	2	3
LS odd	$n' = 0$		-0.57	-0.90	-0.96	-1.06
	1			-1.28	-1.40	-1.55
	2				-1.71	-1.91
	3					-2.19

## 3.2. EFFECTIVE INTERACTION FOR INELASTIC SCATTERING

A least-squares fitting of the effective  $G$ -matrix elements in the various channels to a set of Yukawa potentials with at most four ranges was performed. This was done to facilitate use with the inelastic scattering program DWBA70 [ref. <sup>17</sup>], which can handle a sum of central, tensor and spin-orbit components of the form expressed by eq. (19). The choice of the ranges was physically motivated, as discussed in ref. <sup>1</sup>), and consisted of 0.25, 0.4, 0.7 and 1.414 fm. The one-pion exchange potential strength was imposed on the 1.414 fm part of the central interaction, as required by the meson theory of nuclear forces.

The results are shown in table 2 where, for each channel, two rows of interaction strengths are given. The first row is that derived from the Paris potential, the second that from the Reid potential. The latter numbers are those given in ref. <sup>1</sup>) for the SE, TE and TNE channels; for other channels, they have been obtained *ab initio*. The similarity of the two sets of numbers in most of the channels is noteworthy. It again indicates that the differences that exist between the Paris potential and the Reid potential will not show up to the same extent in inelastic scattering results. We verify this quantitatively in the next section. The central odd components are the most poorly determined parts of the interaction, as reflected in the very different strengths of the Yukawa interactions derived from the Reid and Paris potentials. It is also reflected in the rather small values of the effective  $G$ -matrix elements found for the SO and TO channels (table 1).

TABLE 2

Best-fit interaction strengths (in MeV)

Channel	Name	$R_1 = 0.25$ fm	$R_2 = 0.40$ fm	$R_3 = 0.7$ fm	$R_4 = 1.414$ fm
SE	Paris	11466	-3556		-10.463
	Reid	12455	-3835		-10.463
TE	Paris	13967	-4594		-10.463
	Reid	21227	-6622		-10.463
SO	Paris	-1418	950		31.389
	Reid	29580	-3464		31.389
TO	Paris	11345	-1900		3.488
	Reid	12052	-1990		3.488
TNE	Paris		-1096	-30.9	
	Reid		-1260	-28.4	
TNO	Paris		244	15.6	
	Reid		263	13.8	
LSE	Paris	-5101	-337		
	Reid	-4382	-352		
LSO	Paris	-1897	632		
	Reid	-2918	483		



#### 4. Application

Inelastic scattering cross sections were computed with the microscopic code DWBA70 for various levels excited in the  $^{24}\text{Mg}(p, p')$  reaction. Zwiaglinski *et al.*<sup>5)</sup> have studied this reaction experimentally at 40 MeV and also analyzed it microscopically using a particular choice of the interactions given in ref.<sup>1)</sup>. They used detailed shell-model wave functions from Chung and Wildenthal<sup>18)</sup> for the levels of  $^{24}\text{Mg}$ . We have repeated this analysis, and also performed similar calculations using the two sets of interaction listed in table 2. The shapes of the angular distributions calculated with all three sets of interaction are identical. We do not show them here, as they have already been published in ref.<sup>5)</sup>. The normalizations  $N$ , defined as the ratio  $\sigma_{\text{exp}}/\sigma_{\text{calc}}$ , are similar for the three interactions. Table 3 lists the values of  $N$  obtained for some of the states in  $^{24}\text{Mg}$ . These states have been selected to explore the behavior of the effective interaction in  $T = 0$  and  $T = 1$  channels, for both spin-flip (odd  $J$ ) and non-spin-flip (even  $J$ ) transitions. The departure of  $N$  from unity has been attributed<sup>5)</sup> to core polarization effects. We note that in all these four channels, the different interactions give similar normalizations. Thus the effective interaction for inelastic scattering obtained from the Paris potential is very similar to the interactions derived from the earlier, phenomenological potentials.

This result is consistent with the finding of Chakravarti *et al.*<sup>19)</sup> regarding the effective charge and interaction in the sd shell. They too find that the results from the Paris potential are close to those obtained previously with the Reid soft-core potential.

In conclusion, nuclear structure models favor interactions with very strong exchange, and we had hoped that the new potentials based on better P-wave phase shifts would improve the exchange character of the effective interaction. However, the difference between the older  $G$ -matrix elements and the newer ones is small, and no improvement is found in the effective interaction at the level of the  $G$ -matrix.

TABLE 3  
Normalizations for levels populated in  $^{24}\text{Mg}(p, p')$  at 40 MeV

$J^\pi T$	$N_{\text{Reid}}$	$N_{\text{Paris}}$	$N_{\text{Zwiaglinski}}^{\text{a)}}$
$2^+0$	2.67	2.74	3.89
$3^+0$	4.83	4.95	6.04
$2^+1$	2.64	2.69	3.20
$3^+1$	1.69	1.67	1.82

<sup>a)</sup> Values obtained using the interaction and method of ref.<sup>5)</sup> but with normalizations done by the present authors.

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