This is correct if the hump in the spectrum contains essentially all the states that can be reached by the knock-out of a particular proton [7]. There is evidence that this is a valid assumption for closed shell nuclei [12,13], but that particularly for deformed nuclei final states with high energies of excitation have fairly high probabilities of being reached. These would certainly not be included in the hump in the experimental summed energy spectrum.

The other point concerns the eigenenergies of the single particle wavefunctions that are used to calculate the $\langle T_i \rangle$. This problem is discussed in a recent review of the whole problem of single particle energies in nuclei [13] and it is shown there that these eigenenergies can in fact be identified with the experimental separation energies even when the re-arrangement energy is large, provided it is taken into account. The article also draws attention to the fact that we have at present no reliable estimate for the other kind of re-arrangement energy, due to orbital re-arrangement. It may be argued that this is likely to be least for nuclei with one hole in a closed shell.

We conclude that the simple theory should be used only for closed shell nuclei. Within the accuracy of the theory, the re-arrangement energy then shows no variation with A and a value of 5-6 MeV for infinite matter seems indicated.

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THE 2-PARTICLE TRANSFER REACTION WITH T = 1 PARTICLE-HOLE STATES

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In contract to many types of reactions, the two-particle transfer reaction leading to T = 1 particle-hole states shows an inhibition to the ground state and coherent enhancement to excited states.

The spacial correlations between valence nucleons can have important effects in reactions. We show below for a simple case of a nucleus with closed shells and one neutron and one proton hole, such as ^{40}K or ^{56}Co, that the valence p-h pair will tend to be anticorrelated in the low-lying states. This should be observable in the 2-particle transfer reaction, such as ^{54}Fe (^{4}He,p) ^{56}Co, by the depletion of strength to the low states.

Just the opposite happens with T = 0 p-h pairs. These tend to be strongly correlated in the nucleus, and because of this, near closed shells

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the collective vibrational states also "collect" the 2-nucleon transfer strength [1-3].

In the $T = 1$ state the particle-hole force is repulsive, being given by

$$\overline{V}_{T=1}(\mathrm{ph}) = -\tfrac{1}{2}\overline{V}_{T=1}(\mathrm{pp}) - \tfrac{1}{2}\overline{V}_{T=0}(\mathrm{pp}).$$

Therefore, the correlations that are built up in the ground state have the particle as far away from the hole as possible. From the perspective of the shell model calculations, this has two well-known implications. First, the ground state will not be the state of lowest or highest spin allowed by the coupling of the particle-hole configuration, but rather will have some intermediate spin. This is illustrated by ^{40}K or ^{56}Co, which both can have J vary from 2 to 5 in the lowest configuration, $f_{\frac{7}{2}}d_{\frac{3}{2}}^{-1}$ or $p_{\frac{3}{2}}f_{\frac{7}{2}}^{-1}$. For both of these nuclei, $J = 4$ and 3 are the lowest states.

Second, when more sophisticated wavefunctions for the low-lying states are built by mixing configurations, the phase of the admixed amplitudes are always such as to decrease the probability of the two particles being close together. The magnitudes of the admixed amplitudes are of course determined by the Hamiltonian dynamics.

Despite the large single-particle splitting of the orbitals, the effect is quite strong for $T = 0$ particle-hole pairs (where they are attracted). The $T = 1$ particle-hole force is much weaker, so that the induced anticorrelations will not in general be strong. However, in the region with $N, Z > 28$ there are several orbitals close together ($p_{\frac{3}{2}}$, $f_{\frac{5}{2}}$, $p_{\frac{1}{2}}$), allowing anticorrelations to be produced even with a weak force.

The mechanism whereby the two-particle transfer strength is pushed into the excited states is simple. The stripping reaction places the two particles close together in the nucleus. In the case of ^{54}Fe (^{3}He, p)^{56}Co, the transferred proton fills one of the two proton holes in ^{54}Fe. Since the holes are quite likely to be close together *, the transferred neutron is constrained to be close to the remaining proton hole. This process is depicted in fig. 1. But a neutron near proton hole is unfavored energetically; neutrons attract protons, not proton holes. Thus the states populated in the reaction would tend to lie at high excitations - in sharp contrast to other reactions.

This reasoning was tried out with a DWBA calculation on ^{54}Fe (^{3}He, p)^{56}Co which does indeed predict the above behavior. A few details

* Actually, they are only likely to be on the same axis, since the s.p. parity is almost pure.

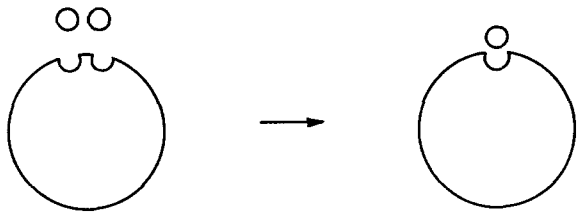

Fig. 1.

of the computation, which is standard, are given below.

To determine the nuclear wavefunctions we used the same particle energies as Vervier [4] and Kuo's two-body matrix elements which include core polarization [5]. This gives an entirely reasonable spectrum and absolute binding. The wavefunctions for the lowest states are given in table 1.

The mixing is strongest in the 2^+ states and weakest in the 5^+.

Having obtained the wavefunctions, the form factor for two-body transfer is calculated according to Glendenning's procedure [6]. First the coordinates of the core nucleons are integrated out of the matrix element

$$\langle\psi(^{54}\mathrm{Fe})|\psi(^{56}\mathrm{Co})\rangle =$$
$$= \sum_j a_j \langle (f_{\frac{7}{2}}^{-1} f_{\frac{7}{2}}^{-1})^0 \varphi(^{56}\mathrm{Ni})|\, (j\, f_{\frac{7}{2}}^{-1})^J \varphi(^{56}\mathrm{Ni})\rangle$$
$$= \sum_j a_j \,|\, (j\, f_{\frac{7}{2}})^J\rangle$$

Next, the relative coordinate of these two particles is integrated out, taking an overlap with the relative coordinate of the transferred pair in the ^{3}He internal wavefunction. This is, in Glendenning's notation,

$$\sum a_j \langle \chi_{\mathrm{np}}(r_\mathrm{p} - r_\mathrm{n})|\, (j\, f_{\frac{7}{2}})^J = \sum_{NL} G_{NLSJ}\, u_{NL}(R)$$

where $u_{NL}(R)$ are oscillator functions of the center of mass coordinate, and the coefficients G_{NL} are determined numerically. Woods-Saxon well wavefunctions were used for the single-particle orbitals. These were expanded in terms of harmonic oscillator wavefunctions to deter-

Table 1

J	Configuration $p_{\frac{3}{2}}f_{\frac{7}{2}}^{-1}$	$p_{\frac{1}{2}}f_{\frac{7}{2}}^{-1}$	$f_{\frac{5}{2}}f_{\frac{7}{2}}^{-1}$
2+	0.893		0.449
3+	-0.951	-0.052	0.306
4+	0.952	0.059	0.298
5+	-0.952		0.306

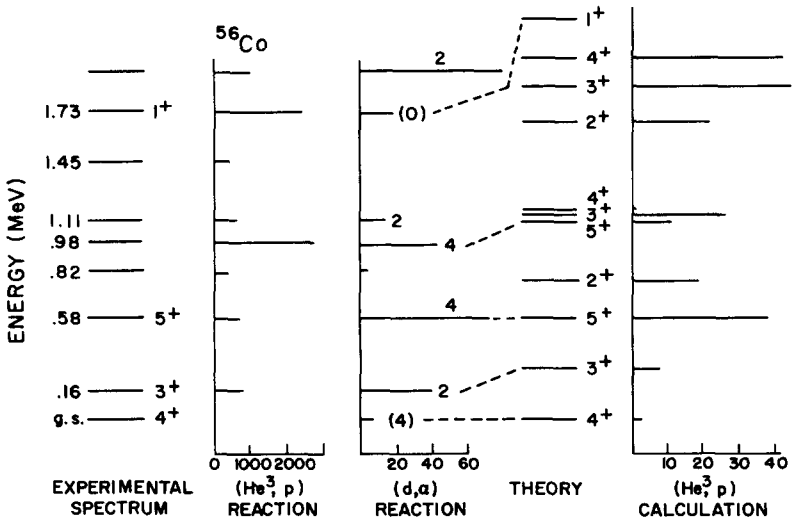

Fig. 2. Comparison of theory with experiment on the 2-particle transfer reaction to ^{56}Co. In the (^{3}He,p) experiment of ref. 7, counts at $\theta = 15^\circ$ are plotted. For the (d,α) experiment of ref. 8, the maximum differential cross-sections in μb/st. is given, along with the l-transfer deduced from the stripping pattern. The theoretical levels were calculated with Kuo's force [5] and have been shifted upward by 0.26 MeV to give the ground state proper binding.
Ploted for the (^{3}He,p) reaction is $(d\sigma/d\Omega)_{max}$, in arbitrary units.

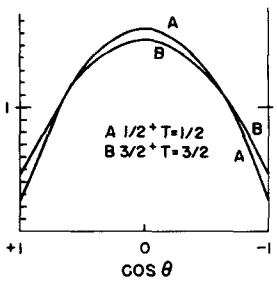

Fig. 3. The hole distribution in 2p-1h states of ^{41}Ca. The probability is plotted of the hole located at a given angle with respects to the particle-particle axis, when the particles lie along the same axis. State A is the lowest $T = \frac{1}{2}$ $J = \frac{1}{2}$ state, and B is the lowest $T = \frac{3}{2}$ $J = \frac{3}{2}$ state from ref. 9.

mine the G coefficients *. As many as 6 terms were included in the sum over N. However, only one or two terms were necessary for calculating the relative cross-sections.

Finally, this form factor was put in the distorted wave program JULIE, giving for a final cross section

$$\frac{d\sigma}{d\Omega} = (2J+1)\left(\tfrac{1}{2}\frac{d\sigma^{(S=0)}}{d\Omega}_{\text{JULIE}} + \tfrac{1}{2}\frac{d\sigma^{(S=1)}}{d\Omega}_{\text{JULIE}}\right)$$

The energy levels, and peak differential cross sections (for 18 MeV ^{3}He particles) are plotted in fig. 2, along with the experimental data.

The correlation effects are quite dramatic, both in the correlation made by coupling to J, and the induced correlation of configuration mixing. Without mixing, the $p_{\frac{3}{2}} f_{\frac{7}{2}}^{-1}$ configuration would have several times the strength of the $f_{\frac{5}{2}} f_{\frac{7}{2}}^{-1}$ configuration. When the states are mixed, in every case the phases give a calcellation for the strength to the lowest state, so that among states of the same spin, the cross section tends to go up with excitation. The 5^+ states didn't get mixed strongly enough for this to happen, but this seems to be true experimentally also.

Data have only been reported at one angle [7] so comparison with experiment is not clear.

* Help by Wm. Gerace on computer-programming here and elsewhere is gratefully acknowledged.

However, a complete analysis has been made on a similar reaction [8], ^{58}Ni (α,d)^{56}Co. Here the qualitative trend is in agreement with the expectation. In particular, the cross section to the ground state is very low. The 3^+ and 5^+ are seen and correspond well with the calculated levels. The calculated strength for the 2^+ state is not observed; either the state lies higher in energy, or the mixing between the two 2^+ states is stronger than calculated. An earlier set of matrix elements, which produced only slightly stronger mixing between the 2^+ states, did in fact give almost complete interference for the transition to the lower state.

We conclude with a suggestion that the anti-correlations of particles and holes might be a more widespread phenomenon, making the (^{3}He,p) reaction difficult to do where different orbitals are being filled simultaneously. The core-polarization corrections to the particle-hole force give it an anticorrelating part in both the $T = 1$ and $T = 0$ particle-hole states. This part of the force is coherent among many particles and many holes, so that the single-particle splitting can be eventually overcome. The nucleus is then deformed or aligned, with the particles along one axis and the holes along a perpendicular axis, and should be well-described by the Nilsson model. It would now be difficult to populate a ground state by transferring two particles, one to each axis. Fig. 3 depicts the wavefunction calculated for 2p-1h states of ^{41}Ca [9]. The tendency for alignment is already quite strong. A typical reaction which should illustrate this is ^{40}Ar (^{3}He, p)^{42}K.

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ON THE NUCLEAR OCTUPOLE DEFORMATION IN THE $218 \leq A \leq 232$ REGION

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The potential energy of a nucleus is expressed as a function of parameters of the quadrupole and octupole deformation. The minimum for all nuclei in the given region appears at $\beta_3 = 0$ i.e. there exist no nuclei with a stable octupole deformation. An assymetry is observed of the $E(\beta_2, \beta_3)$ surface towards the $\beta_2 = \beta_{\text{equilibrium}}$ plane which should be interpreted as an interaction of β-vibrations and octupole vibrations.

The dependence of potential energy of nuclei on the octupole deformation parameter is in particular important for the explanation of properties (energies, moments of inertia, hindrance factors of α-decay etc.) of the lowlying negative parity states. Such states appear systematically in even-even nuclei in the $218 \leq A \leq 232$ region [e.g.1] and the references metioned there). The most natural way of explanation is to consider these states as octupole harmonic vibrations. The respective calculations [2] performed by the RPA method gave for most of the nuclei of the actinide and rare earth regions good agreement with the experiment. They led, however, to imaginary energies for nuclei with $A < 228$. The question of nuclear stability with respect to octupole deformation was investigated in [3-5]. By various approximations (usually perturbation theory) the conclusion was reached that in the region considered nuclei are "soft", but do not have stable octupole deformation.

The purpose of this work is to obtain more detailed information about the dependence of potential energy on the parameters of quadrupole and octupole deformations. We shall use the method which already has proved to be successful in considering the quadrupole deformations [6,7]. In this method the energy of a nucleus is taken as the sum of energies of independent nucleons moving in the deformed field. The pairing correlations do not effect the position of energy minimum considerably, usually causing only a "flattening" of the corresponding curves. Therefore we shall neglect them in our calculations as well as the Coulomb interaction which for small deformations has not a great influence.

Let the equipotential surfaces of the average field be axially symmetric and have the form

$$r = R_0(1 + \beta_2 Y_{20}(\theta,\varphi) + \beta_3 Y_{30}(\theta,\varphi) + - \frac{9}{2}\sqrt{\frac{3}{35\pi}}\,\beta_2,\beta_3 Y_{10}(\theta,\varphi)).\, k(\beta_2,\beta_3), \tag{1}$$

where $k(\beta_2, \beta_3)$ assures the conservation of volume. In analogy to the Nilsson potential [8] we obtain the one-particle Hamiltonian in the lowest approximation in β_2, β_3 in the form

$$H = -\frac{\hbar^2}{2M}\Delta + \frac{1}{2}M\omega_0^2 r^2\left[1 + \frac{5}{4\pi}(\beta_2^2 + \beta_3^2)\right] \times \left[1 - 2\beta_2 Y_{20} - 2\beta_3 Y_{30}\right] + C\cdot l\cdot s + D\cdot(l^2 - \langle l\rangle^2_{\text{shell}}) \tag{2}$$

The octupole potential of the $\sim r^3 Y_{30}$ type em-