Pauli Suppression of Momentum Fluctuations

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Pauli correlations are shown to be important in interpreting the momentum distribution of nuclear fragments formed in high-energy heavy-ion collisions.

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The fast fragments emerging from relativistic heavy-ion collisions have a narrow momentum distribution with a Gaussian shape.[1] This distribution is described with some success by the independent-particle model.[2] However, the independent-particle model ignores all correlations beyond the purely kinematical ones. The Pauli correlations, in particular, are very important and reduce the dispersions in measurements of one-body operators. I will show that this is also the case for the momentum operator, and that inclusion of these correlations improves agreement between theory and experiment.

I first recall Goldhaber's derivation of the momentum dispersion in the independent-particle model.[2] One can pick F nucleons from the projectile to make the fragment, and calculate the expectation of the squared momentum of these nucleons in the projectile initial state. In the frame of the projectile, this yields a dispersion

$$\sigma^2 = \langle [\sum_i^F p_z(i)]^2\rangle$$
$$= F\langle p_z^2(1)\rangle + F(F-1)\langle p_z(1)p_z(2)\rangle . \quad (1)$$

The first term in Eq. (1) can be estimated in the Fermi-gas model as[3]

$$\langle p_z^2\rangle = \tfrac{1}{5} p_F^2,$$

where p_F is the Fermi momentum, as measured, for example, by inelastic electron scattering.

To calculate the second term in Eq. (1), Goldhaber uses the fact that the total momentum of the original nucleus is zero:

$$\langle [\sum_i^A p_z(i)]^2\rangle$$
$$= A\langle p_z^2(1)\rangle + A(A-1)\langle p_z(1)p_z(2)\rangle = 0. \quad (2)$$

Combining Eqs. (1) and (2), I find, for the momentum dispersion,

$$\sigma^2 = \langle(\sum_i^F p_i)^2\rangle = [F(A-F)/(A-1)](\tfrac{1}{5}p_F^2). \quad (3)$$

Experimentally, with ^{40}Ar projectiles, it is found that the dispersion σ^2 is 31% lower than predicted by Eq. (3).[4]

I will improve this model by treating the correlation terms involving identical nucleons more carefully. Qualitatively, one expects that there is a large anticorrelation between the momenta of two identical particles when they are close together in coordinate space. To calculate the effect, one needs to include some spatial details of the measurement process. The experiment measures the momentum in the z direction when a region of matter delineated in the transverse directions is removed from the projectile. Thus the operator acting on the nucleus has the form

$$\mathcal{O} = p_z f(x,y), \quad (4)$$

where $f=0$ or 1 in the regions of the nucleus that are removed or remain in the fragment. The operator $\mathcal{O}^2$ can be easily evaluated in a determinantal wave function that is separable in Cartesian coordinates,

$$\psi = \mathcal{A}\prod_n^a \varphi_{n_x}(x)\varphi_{n_y}(y)\varphi_{n_z}(z). \quad (5)$$

Here $n\equiv(n_x, n_y, n_z)$ labels the quantum numbers of the occupied orbits, and the total number of orbits is a. The matrix elements are evaluated as

$$\langle\psi|\sum_i^a \mathcal{O}_i^2|\psi\rangle = \sum_n \langle n_x n_y|f|n_x n_y\rangle\langle n_z|p_z^2|n_z\rangle, \quad (6)$$

$$\langle\psi|\sum_{i\neq j}^{a(a-1)} \mathcal{O}_i\mathcal{O}_j|\psi\rangle$$
$$= -\sum_{n\neq n'}\langle n_x n_y|f|n_x' n_y'\rangle^2\langle n_z|p_z|n_z'\rangle^2. \quad (7)$$

I also need to include the correlation between nonidentical nucleons. The complete expression for the momentum dispersion in a nucleus with four nucleons in each orbit, i.e., $A=4a$, is given by

$$\langle\mathcal{O}^2\rangle = 4\langle\psi|\sum_i^a \mathcal{O}_i^2|\psi\rangle + 4\langle\psi|\sum_{i\neq j}^{a(a-1)}\mathcal{O}_i\mathcal{O}_j|\psi\rangle + 12a^2\langle\mathcal{O}_i\mathcal{O}_k\rangle_{\text{nonidentical}}. \quad (8)$$

In the last term, the expectation value is an average over particles having different spin and isospin quantum numbers.

It can be shown that in the limit of a very large nucleus, the first two terms in Eq. (8) already reduce the dispersion by a factor of the order of $A^{1/3}/\ln A$ (Ref. 5). However, for quantitatively useful results one needs to evaluate Eq. (8) more explicitly. I use harmonic-oscillator wave functions to evaluate the first two terms. The last term I obtain analogously to Eq. (2). Neglecting the spatial correlations between nonidentical nucleons,

$$12a^2\langle \mathcal{O}_i \mathcal{O}_j \rangle_{\text{nonidentical}} \cong \frac{3F^2}{4}\langle p_i p_j \rangle_{\text{nonidentical}} = -\frac{\langle 0|p_z^2|0\rangle}{3a}\frac{3F^2}{4}. \tag{9}$$

The last step follows from Eq. (8) for the full nucleus, $f=1$. I now take $f=\theta(x)$, which leaves half the nucleus in the fragment. For $a=10$, corresponding to ^{40}Ca, this yields for the three terms in Eq. (8),

$$\langle \mathcal{O}^2\rangle = (40-17.0-10)\langle 0|p_z^2|0\rangle = 13.0\langle 0|p_z^2|0\rangle.$$

To compare with the independent-particle model, I note that for $a=10$, $\langle p_z^2\rangle = 2\langle 0|p_z^2|0\rangle$, and

$$\langle \mathcal{O}^2\rangle_{\text{independent particle}} = [20(20)/39]2\langle 0|p_z^2|0\rangle = 20.5\langle 0|p_z^2|0\rangle.$$

Thus the dispersion is reduced by 37%, compared to 31% measured reduction. For a very heavy nucleus, such as ^{238}U, the calculated reduction in the dispersion is more than a factor of 2. It would be interesting to confirm the A dependence predicted by the Pauli correlations.

Before asserting that the effect of Pauli correlations has been demonstrated, I must examine other processes that affect the momentum distribution.[6] Particles evaporating at temperatures ~1 MeV do not contribute significant momentum, but do affect the final fragment mass. However, Eq. (3) is insensitive to the fragment mass near $F=A/2$, so the presence of evaporation of ~5 particles would not affect the conclusion. Other dynamics effects beyond the initial state correlations would increase σ^2, and so make the presence of the Pauli reduction more compelling.

It would also be interesting to study the effect of the Pauli correlations on the transverse-momentum distribution. Experimentally, it is the same as the longitudinal-momentum distribution at relativistic energies.[1] Since the fragmentation operator produces a spatial correlation in the transverse directions, I expect the momentum anticorrelations to be enhanced transversely. Unfortunately, the operator itself introduces transverse momentum; a calculation of the net effect requires a more detailed model, which is beyond the scope of this Letter.

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