THE NUCLEAR DENSITY OF STATES IN THE SPACE OF NUCLEAR SHAPES

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The density of states for the nuclear shape degrees of freedom are calculated in the Fermi gas model. For quadrupole deformations, the resulting formulas agree well with the properties of the deformed excited states of ^{16}O and ^{40}Ca. Application is also made to the inertia associated with the deformation coordinate. The inertia turns out to be much smaller than given by the weak coupling limit.

The nuclear shape degrees of freedom introduce a complexity into the density of states that has not been adequately characterized. In this note we will address the question of how dense the population of lowest states is in the space of nuclear shape degrees of freedom.

The lowest states may be defined by constrained Hartree–Fock calculations. For example, consider some particular shape degree of freedom parameterized by a coordinate y, with the Hartree–Fock states constrained to have a shape moment corresponding to y. Then the lowest state would lie on a set of parabolas as shown in fig. 1. Each parabola defines a different Hartree–Fock state, and the question is to determine the number of states per unit interval in y, dn/dy.

It is first necessary to understand how the nuclear wavefunction depends on deformation. For small changes in y, the Hartree–Fock state remains in the same parabola, and the transformation of the wavefunction is well described by a displacement field $\boldsymbol{u}(\boldsymbol{r})$,

$$\psi_{y+\Delta y}(r_1, r_2, \ldots) = \psi_y(r_1 + \Delta y\, \boldsymbol{u}, r_2 + \Delta y\, \boldsymbol{u}, \ldots) . \quad (1)$$

Because it requires more energy to compress nuclear matter than to deform it, the transformation will be volume preserving, $\nabla \cdot \boldsymbol{u} = 0$. For example, for quadrupole deformations of spherical nuclei, the displacement field will have the form [‡1]

$$\boldsymbol{u} = \nabla r^2 P_2(\hat{r}) . \quad (2)$$

An important effect of the transformation (1) is to modify the local momentum of the nucleons according to [1]

$$p' = p \cdot (1 - \Delta y \nabla \boldsymbol{u}) . \quad (3)$$

This results in a distortion of the Fermi surface. The extra kinetic energy of the distortion in fact provides the restoring force toward the equilibrium of the parabola.

The equilibrium is characterized by a spherical Fermi surface, and this provides the criterion for how much distortion is required to pass from one equilibrium y_1 to the next, y_2. On the average, a new minimum is possible when the volume of phase space in the distorted Fermi surface with $p > p_F$ exceeds $(2\pi)^3$, i.e.,

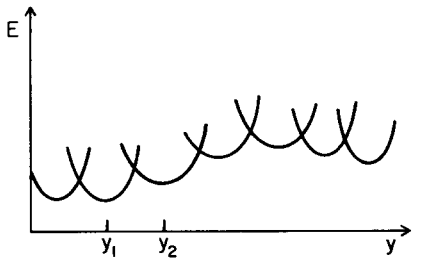

Fig. 1. Schematic picture of Hartree–Fock energy as a function of shape parameter.

[‡1] This may be seen from the Wigner representation of the density matrix, making a Taylor series expansion of $\boldsymbol{u}$. Cf. ref. [1].

the phase space for one particle. Then the system can achieve a new spherical momentum distribution by moving the particle from above p_F to the empty volume below.

To quantify these observations, we make a principal axis transformation of the tensor ∇u, and assume axial symmetry:

$$\nabla u = \begin{pmatrix} a(r) & 0 & 0 \\ 0 & -\frac{1}{2}a(r) & 0 \\ 0 & 0 & -\frac{1}{2}a(r) \end{pmatrix} . \quad (4)$$

The number of particles with $p > p_F$ is then

$$n = \Delta y \int_{\text{nucleus}} d^3r\, a(r) \times \int_{1/\sqrt{3}}^{1} 4\pi\, d\cos\theta \left(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}\right) \frac{p_F^3}{(2\pi)^3}$$

$$= \Delta y\, \rho_0 \int_{\text{nucleus}} d^3r\, a(r)/\sqrt{3} , \quad (5)$$

where ρ_0 is the density of the nucleus.

We first apply this to quadrupole distortions of spherical nuclei. For the deformation coordinate it is convenient to replace y by the more familiar variable β,

$$\beta = 2y\sqrt{4\pi/5} .$$

With the displacement field (2), $a(r) = 2$. The integration in eq. (5) is then trivial, yielding

$$n = \beta\sqrt{5/12\pi}\, A , \quad (6)$$

where A is the mass number of the nucleus. The nuclei ^{16}O and ^{40}Ca have prominent deformed states in their spectra, and their properties can be compared with the prediction eq. (6). In these nuclei, there is a high degree of spin–isospin symmetry. The interaction energy can only be maintained with each orbit occupied by four particles. Putting $n = 4$ in eq. (6), we obtain the theoretical β values for the deformed states shown in table 1. These are compared with empirical values determined from the experimental [2,3] $B(E2)$'s, according to the formula [4]

$$\beta = \frac{3}{4(0.945)} \frac{[16\pi B(E2\downarrow)]^{1/2}}{Z\langle r^2\rangle}$$

Table 1
Characteristics of deformed states in ^{16}O and ^{40}Ca.

Nucleus	β		Surface energy eq. (7) (MeV)	Experimental excitation energy (MeV)
	eq. (6)	exp		
^{16}O	0.70	0.84	9	6.06
^{40}Ca	0.27	0.26	2.5	3.35

Since the Fermi surface is again spherical in the deformed state, the energy difference should be due to surface energy only. Table 1 also compares the excitation energy with the additional surface energy of the deformed state according to the liquid drop formula [3],

$$V_{\text{surf}} = 2\beta^2 R_0^2 S , \quad S \approx 1\ \text{MeV/fm}^2 . \quad (7)$$

We see that while ^{16}O is perhaps too small for such a crude model to work well, the agreement is satisfactory for ^{40}Ca.

Next we consider the dynamics of fission of heavy nuclei. In macroscopic treatments [5], the dynamic equations are based on a parameterization of the effective mass of the fluid. In microscopic terms, the fission proceeds by passing through one Hartree–Fock state after the other. To make a connection between these two treatments, it is necessary to estimate dn/dy for the fission coordinate. We use eq. (6) with $n = 2$, since the nucleus moves from one state to the next by a pairing interaction. For the nucleus ^{240}Pu, we then find for the average distance between two configurations

$$\Delta\beta = \frac{2}{(240)} \sqrt{\frac{12}{5}} \approx 0.023 . \quad (8)$$

To compare this with the actual Hartree–Fock density of states, we count the level crossings along the deformation coordinate in ref. [6]. From their figs. 7 and 8, we find 29 states in an interval $0 \leqslant y \leqslant 0.2$. With the relation for their coordinate y, $y \approx 0.27\,\beta$, this implies an average distance $\Delta\beta = 0.025$, in good agreement with eq. (8).

The kinetic energy for a collective hamiltonian in a deformation coordinate y may be easily derived in the weak coupling approximation, in which it is assumed that only neighboring Hartree–Fock states are coupled

by the residual interaction. Then the hamiltonian matrix is tridiagonal and the comparison with the tridiagonal matrix that represents the kinetic energy operator on a coordinate mesh yields

$$T_y^{\text{weak coupling}} = \frac{\partial}{\partial y}\left(\frac{dy}{dn}\right)^2 v_0 \frac{\partial}{\partial y}. \quad (9)$$

Here v_0 is the strength of the residual interaction. This is to be compared with the hamiltonian based on the classical irrotational inertia, which for β deformations is given by

$$T_\beta^{\text{classical}} = \frac{\partial}{\partial \beta}\frac{4\pi}{5Am\langle r^2\rangle}\frac{\partial}{\partial \beta}. \quad (10)$$

These have quite different functional forms. However, we can compare in given cases. If we evaluate these expressions for ^{240}Pu, using a typical pairing strength $v_0 = 0.1$ MeV in eq. (9), we find

$$T_\beta^{\text{classical}}/T_\beta^{\text{weak coupling}} = 150 .$$

According to detailed Hartree–Fock–Bogoliubov calculations [7], the actual factor is of the order of 10. So while the pairing is not strong enough to make the classical T a good approximation, it is strong enough to invalidate the weak coupling. Effectively, v_0 in eq. (9) needs to be increased to the order Δ, the pairing gap.

These rough considerations ought to be useful for the treatment of other problems where more quantitative techniques have been too difficult to apply. One such problem is the dispersion in masses of nuclei emerging from heavy ion collisions. The time-dependent Hartree–Fock calculations show that without a dispersion in shapes there is very little dispersion in mass [8]. The theory needs to include explicitly the shape degrees of freedom and the residual interaction which connects different shapes. Another problem is understanding the high density of levels at excitations $\approx$10 MeV. These are underpredicted in the Fermi gas model, which of course ignores the shape degrees of freedom [9]. From microscopic calculations [10], the quadrupole deformations are seen to be important at these excitation energies. What is needed for a better treatment of level densities is to combine the description of phase space for the deformed ground states with the Fermi gas model for intrinsic excitations.

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