which, in principle, this density distribution was allowed to feed back by deforming the well (5). It is clear that then the linear dimension in the z direction will increase, so that c increases and c is just the term in (11) with the highest weight.

We now mention the numerical values of the matrix element M for different current deformations. We use a value d = 1.78 fm which corresponds to $\hbar\omega$ = 13.1 MeV.

With pure SU_3 wave function one gets $a = b = c = d$, M = 1.1 fm^2. With Brown's axially symmetric deformations for the 2p2h state (β = 0.3) and the 4p4h state (β = 0.5) one gets M = 2.6 fm^2. With Hayward's best Hartree-Fock wave functions one finds $a = 0.89\ d$, $b = d$, $c = 1.12\ d$ for the 2p2h state and $a = 0.80\ d$, $b = d$, $c = 1.26\ d$ for the 4p4h state which leads to M = 2.8 fm^2. If one, in the last case, assigns to the 2p2h state the same deformation as the 4p4h state one even finds M = 3.4 fm^2.

Although the experimental value of 3.8 fm^2 is still higher, Brown's wave functions seem to be a little better than those of Wong [4] who considers a linear combination of 0p0h, 1p1h and 2p2h wave functions and finds values $M \approx 5$ fm^2, but much too small E2 transition probabilities.

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MONOPOLE MATRIX ELEMENT IN ^{16}O

G. F. BERTSCH
Niels Bohr Institute, Copenhagen

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The strong hindrance of 2γ decay in the monopole decay of ^{16}O is explained with the wave functions of Brown and Green.

The transition amplitude in the decay of the first excited 0^+ state of ^{16}O to the 0^+ ground state is of the order of the single-particle transition amplitude for decay by e^+e^- pair production, but is very small for deacy by $\gamma\gamma$ production. The single-particle estimate requires in a microscopic theory that a one-particle one-hole excitation dominate the 0_2^+ wave function. However, it is difficult to see how there could be much even-parity one-particle one-hole strength as low as the 6.06 MeV 0_2^+ state. Shell-model calculations show that 1-P 1-H states do not mix strongly with many-particle collective excitations [1]. Furthermore, as emphasized by Eichler and others [2], the single-particle model predicts that the 2γ production amplitude is comparable to the e^+e^- amplitude.

Reasonably large e^+e^- pair production matrix elements can also be obtained from theories of wavefunctions based on many-particle excitations and ground-state correlations. In an early attempt to understand E0 transitions, Schiff [3] used wavefunctions which mixed the shell-model ground state with a two-particle two-hole excitation. More recent wavefunctions of oxygen assume a deformed wavefunction for the excited state, which can mix strongly with the shell-model ground state by virtue of the low excitation energy [4]. It is characteristic of these wavefunctions, that the nuclear part of the e^+e^- production matrix element,

$$M_{e^+e^-} = \sum_{\text{protons}} r_i^2$$

is large and is enhanced by higher order corrections to the wave function, while the matrix element responsible for 2γ decay, is strongly inhibited. The following calculation uses the wave functions of Brown and Green, augmented by core excitations determined in perturbation theory [5].

In lowest order, the 0^+ wave functions are taken to be a mixture of the shell-model state, and the 2P-2H, 4P-4H states of maximum deformation, conveniently written as

$$|2P\text{-}2H\rangle = \frac{1}{a}\int d\Omega R_\Omega |z^2(1)z^2(2)\bar{x}(1)\bar{x}(2)\rangle$$

$$|4P\text{-}4H\rangle = \frac{1}{a'}\int d\Omega R_\Omega |z^2(1)z^2(2)z^2(3)z^2(4)\bar{x}(1)\bar{x}(2)\bar{x}(3)\bar{x}(4)\rangle .$$

Here $|z(i)\rangle \equiv a_z^+(i)|\rangle$ is the harmonic-oscillator creation operator adding a quantum of excitation to the wave function of particle i. The Hill-Wheeler integral projects out of the intrinsic state the spherically symmetric component, which is normalized to unity by the factor $1/a$ (ref. 6). Fortunately, it is not necessary to use the projected wave functions because of the scalar nature of the operators considered. There is a further simplification, that there are no cross terms in the matrix element connecting different intrinsic states. This is trivial to show for the $\Sigma_i r_i^2$ operator, since a one-body operator cannot change the number of particle-hole pairs by two. The $M_{\gamma\gamma}$ operator contains two-body components, but since in the harmonic oscillator model

$$r_x = \frac{1}{\sqrt{2}}(a_x^+ + a_x)$$

the operator cannot create or annihilate excitations of the form $|z^2\bar{x}\rangle$.

Thus in both cases the total matrix element may be expressed in terms of the diagonal intrinsic matrix elements, numerically,

$$\langle 0_1^+|M|0_2^+\rangle = -0.229\langle|M|\rangle + \\ +0.107\langle 2P\text{-}2H|M|2P\text{-}2H\rangle + 0.122\langle 4P\text{-}4H|M|4P\text{-}4H\rangle . \quad (1)$$

To first order, $M_{e^+e^-}$ is directly proportional to the number of harmonic oscillator quanta in the wave function:

$$\langle M\rangle = N_{\text{proton}}/\nu$$

where $E_0 = N\hbar\omega$ and $\nu = m\omega/\hbar^2\ \text{fm}^{-2}$.

Table 1 shows the value of $M_{e^+e^-}$ determined in this way, with $\hbar\omega$ = 14.5 MeV.

An augmented set of intrinsic wave functions includes polarization of the core orbitals, mixing two major shells. Thus the 0p orbital is replaced by a linear combination of 0p, 1p, and 0f, and the 0d-1s orbitals are augmented by 0g, 1d and 2s. The amplitudes were determined with the force used by Kallio and Kolltveit [7]. Expressing the orbitals in a Cartesian basis, the main effect of the polarization is to carry $|z\rangle$ to $|z^3\rangle$. There are two types of terms enhancing the $M_{e^+e^-}$ matrix element, the off-diagonal terms proportional to the amplitude of excitation in the single-particle wave function such as

$$\langle z|r^2|z^3\rangle = \frac{1}{\nu}\sqrt{\frac{3}{2}}$$

and the larger expectation value of r^2 in the higher shell wave function. In the 2P-2H wave function, 26% of the enhancement comes from the increased probability of the excited core, and in the 4P-4H wave function, 85% is due to this effect. With this high percentage from the excited state, it may no longer be good approximation to restrict the single-particle orbitals to a basis of two shells. The changes in the $M_{e^+e^-}$ caused by a 50% decrease in the matrix element $\langle 1s||v||2s\rangle$ and neglect of the matrix element $\langle 2s||v||3s\rangle$ or an addition of a p-state repulsion of the form $\langle 0p||v||1p\rangle$ = 1.17 MeV (ref. 9), are + 7% and -17% respectively. Core polarizations have also been calçulated with non-singular forces and a variational technique. Using these large polarizations, the E0 matrix element is found to be in much better agreement with experiment [10].

The $\gamma\gamma$ matrix element is proportional to the total E1 strength of the intrinsic states,

$$M_{\gamma\gamma} \sim \sum_n |\langle \text{intrinsic } 0^+|\sum_{\text{protons}} r_i|n\,1^-\rangle|^2$$

since there are only diagonal terms in equation (1). The essential point is that these states have the same E1 strength, although of course they have different expectation values of r^2. This may be seen by noting that the E1 operator is idential to the center-of-mass position operator, except for isospin. All T = 0 states with the center of mass in the harmonic oscillator ground state, have the same R strength and therefore the same E1 strength.

Table 1

The matrix element $\langle 0_1^+|\sum_{\text{protons}} r_i^2|0_2^+\rangle$ with the valence wave function of Brown and Green.

Without core polarization	Perturbed single-particle orbitals	J.Hayward's single-particle orbitals (ref. 10)	Experimental (ref. 11)
1.0 fm^2	1.6 fm^2	2.8	3.8

The matrix element is not identically zero when higher order corrections are taken into account. The core polarizations are not as effective in this case as they were in the r^2 matrix element, because the most important off-diagonal term in the single-particle matrix element,

$$\langle z \mid r_z \mid z^2\rangle\langle z^2 \mid r_z \mid z^3\rangle$$

is partially excluded in the intermediate state. Including the core polarization gives a ratio of the two types of matrix element,

$$M_{\gamma\gamma}/M_{e^+e^-} \sim 0.064$$

as compared to the experimental [11] limit,

$$M_{\gamma\gamma}/M_{e^+e^-} < 0.071.$$

Here the formulae of Oppenheimer and Schwinger [12] has been used to extract the matrix element ratio from the branching ratio, taking the intermediate state energy denominator as E_{dipole} = 22 MeV. One assumption necessary for the near vanishing of $M_{\gamma\gamma}$ is that the intermediate energy for the dipole states be the same for all the intrinsic states. A violation of this by 10% could change the ratio of matrix elements by 150%.

Similar theory should apply to the hindrance of 2γ decay in ^{90}Zr (ref. 8), at least with the valence model of Bayman, Reiner and Sheline [13], because here also there is no crossover $\gamma\gamma$ matrix element connecting the shell-model states, $(p_{\frac{1}{2}}p_{\frac{1}{2}})^0$ and $(g_{\frac{9}{2}}g_{\frac{9}{2}})^0$.

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