SHELL-MODEL CALCULATION OF THE ^{49}Sc SPECTRUM

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The single-particle states of ^{49}Sc are strongly affected by low lying 2p-1h excitations of the structure $\left(\overset{n}{p_{\frac{3}{2}}}\left[\overset{p}{f_{\frac{7}{2}}}\overset{n^{-1}}{f_{\frac{7}{2}}}\right]_L\right)_J$. These 2p-1h states are not very much higher in energy than the $p\frac{3}{2}$ single-particle states.

The nuclei ^{49}Ca and ^{49}Sc both have a single particle outside the double closed shell of ^{48}Ca. Thus, we expect low lying single-particle states in these nuclei. In ^{49}Ca the $p\frac{3}{2}$ ground state and the $p\frac{1}{2}$ first excited state are also found to be rather pure single-particle states, but in ^{49}Sc only the $f\frac{7}{2}$ ground state is pure. The strength of the $p\frac{3}{2}$ state in ^{49}Sc is divided up between two or more states and the $p\frac{1}{2}$ and $f\frac{5}{2}$ strengths are even more spread out. The greater complexity of the ^{49}Sc spectrum might arise because the energy required for 2 particle-1 hole excitations is much smaller in ^{49}Sc than in ^{49}Ca. The lowest 2p-1h states of spin $\frac{3}{2}$ will be of the form

$$\left[\overset{p}{f_{\frac{7}{2}}}\left\{\overset{n^{-1}}{f_{\frac{7}{2}}}\ \overset{n}{p_{\frac{3}{2}}}\right\}_J\right]_{\frac{3}{2}}$$

or, in a more convenient notation

$$\left[\overset{n}{p_{\frac{3}{2}}}\left(\overset{p}{f_{\frac{7}{2}}}\overset{n^{-1}}{f_{\frac{7}{2}}}\right)_L\right]_{\frac{3}{2}}$$

with $L = 1$, 2 and 3. These configurations will not be appreciably higher in energy than the $p\frac{3}{2}$ single-particle state, because the energy required to lift the neutron from the $f\frac{7}{2}$ orbit to the $p\frac{3}{2}$ orbit is partly gained by taking down the proton from the $p\frac{3}{2}$ orbit to the $f\frac{7}{2}$ orbit. 2p-1h states of a similar structure but coupled to spin $\frac{1}{2}$ and $\frac{5}{2}$ mix with the $p\frac{1}{2}$ and $f\frac{5}{2}$ single-particle states.

Information on the ^{49}Sc spectrum comes mainly from the stripping reaction ^{48}Ca(^{3}He,d)^{49}Sc [1,2] and the β-decay of ^{49}Ca to various excited states in ^{49}Sc [3]. Thus, a shell-model calculation can be checked by comparing both energies, spectroscopic factors and β-decay matrix elements to experimental values.

In the shell-model calculation we take the diagonal matrix elements from experimental values and calculate the non-diagonal ones by means of the Kallio-Kolltveit force [5]. Details of the calculation are given in the appendix. The single-particle binding energies are obtained from the stripping data, and the binding energies for the 2p-1h configurations

$$\left[\overset{n}{p_{\frac{3}{2}}}\left(\overset{p}{f_{\frac{7}{2}}}\overset{n^{-1}}{f_{\frac{7}{2}}}\right)_L\right]_J$$

are obtained from the binding of the $p\frac{3}{2}$ neutron in ^{49}Ca and from the low-lying states in ^{48}Sc, which are of the structure

$$\left(\overset{p}{f_{\frac{7}{2}}}\overset{n^{-1}}{f_{\frac{7}{2}}}\right)_L .$$

The remaining very small part of the 2p-1h binding energy, which arises from the interaction between the $p\frac{3}{2}$ neutron and the configuration

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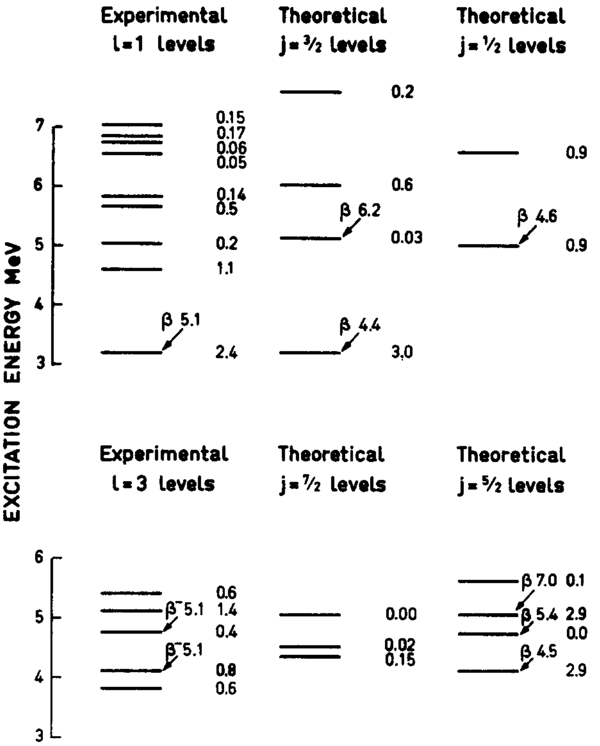

Fig. 1. Comparison of theoretical and experimental properties of ^{49}Sc levels. The spectroscopic factors $(2J+1)S$ measured in the reaction ^{48}Ca(^{3}He,d)^{49}Sc are shown together with the log ft value of the β-decay from ^{49}Ca, where this has been observed (ref. 3). Only the lowest theoretical and experimental levels are shown.

$$\left(f_{\frac{7}{2}}^{p}\, f_{\frac{7}{2}}^{n-1}\right)_L ,$$

is calculated by means of the Kallio-Kolltveit force.

The resulting energies, spectroscopic factors and log ft-values are compared to the experimental values in fig. 1. The experimental findings are not quantitatively reproduced. However, the large retardation of the β-decays to the $\frac{3}{2}$ states comes out qualitatively in the right manner. The calculation gives a hindrance factor of 8 as compared to single-particle values, whereas the experimental hindrance factor is about 30. The retardation is due to the configurations

$$\left[p_{\frac{3}{2}}^{n} \left(f_{\frac{7}{2}}^{p}\, f_{\frac{7}{2}}^{n-1}\right)_1 \right]_{\frac{3}{2}}$$

and

$$\left[p_{\frac{3}{2}}^{n} \left(f_{\frac{5}{2}}^{p}\, f_{\frac{7}{2}}^{n-1}\right)_1 \right]_{\frac{3}{2}}$$

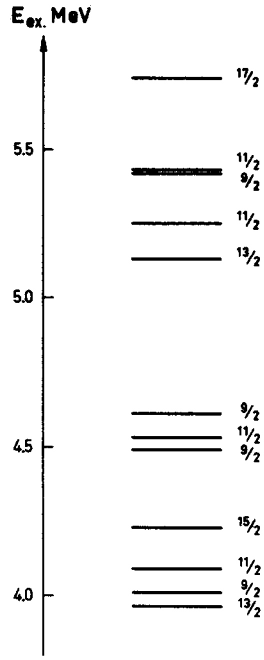

Fig. 2. Low-lying high spin states with the 2p-1h configuration $\sum_L a_L \left[p_{\frac{3}{2}}^{n} \left(f_{\frac{7}{2}}^{p}\, f_{\frac{7}{2}}^{n-1}\right)_L \right]_J$. These states might possible be seen in more complicated reactions such as ^{46}Ca (α,p) ^{49}Sc.

which have large Gamow-Teller matrix elements with the $p\frac{3}{2}$ neutron state in ^{49}Ca, but come in with signs so as to partly cancel the single-particle Gamow-Teller matrix element [4]. The β-transitions to the $\frac{5}{2}$ states are determined only by the 2p-1h configurations

$$\left[p_{\frac{3}{2}}^{n} \left(f_{\frac{7}{2}}^{p}\, f_{\frac{7}{2}}^{n-1}\right)_1 \right]_{\frac{5}{2}}$$

and

$$\left[p_{\frac{3}{2}}^{n} \left(f_{\frac{5}{2}}^{p}\, f_{\frac{7}{2}}^{n-1}\right)_1 \right]_{\frac{5}{2}} ,$$

because the $f\frac{5}{2}$ single-particle states have no Gamow-Teller matrix element with the $p\frac{3}{2}$ neutron state in ^{49}Ca.

The calculated energies and spectroscopic factors only change very little by including more 2p-1h states in the calculation or by replacing the Kallio-Kolltveit force by the Hamada-Johnston potential. As can be seen from the calculated spectroscopic factors, the 2p-1h configurations are of importance in splitting the single-particle strength. However, they do not split it as much or as finely as observed experimentally.

We conclude that single-particle states plus 2p-1h configurations do not provide a sufficient basis for a realistic description of ^{49}Sc, although qualitative trends away from the simple shell-model behaviour are affirmed.

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Appendix. We give here some of the numerical details of the calculation. The single-particle energies, determined empirically, are

	^{49}Sc	^{49}Ca
$f_{\frac{7}{2}}$	9.62 MeV	
$p_{\frac{3}{2}}$	5.80	5.14 MeV
$f_{\frac{5}{2}}$	4.95	1.19
$p_{\frac{1}{2}}$	3.58	3.11

For the $p_{\frac{3}{2}}$ single-particle energy in ^{49}Sc a small change have been made in order to bring the energy of the lowest $\frac{3}{2}$-state in agreement with experiment.

The $\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_L$ interaction can be partially determined from the experimental ^{48}Sc spectrum [6], but for some of the configurations we have to rely on calculations from the ^{42}Sc spectrum [7]. The interaction used was

L	0	1	2	3	4	5	6	7
$V\left(f_{\frac{7}{2}}f^{-1}_{\frac{7}{2}}\right)_L$	≡ 0	-3.38	-5.39	-5.95	-6.25	-6.40	-6.50	-5.18

The interaction matrix elements for the different configurations considered (single-particle, 2p-1h of the form

$$\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_L\right]_J \quad \text{or} \quad \left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{5}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_1\right]_J$$

are expressed in terms of two-body interactions with Racah recoupling coefficients.

All of the matrix elements are obvious, except the off-diagonal one connecting a single-particle state with a 2p-1h state:

$$\langle j(T=\tfrac{7}{2})|V|\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_L\right]_J\rangle =$$

$$= 3\sum_K U(\tfrac{3}{2}\tfrac{7}{2}j\tfrac{7}{2};KL)\,U(j\tfrac{7}{2}j\tfrac{7}{2};K0)\times$$

$$\times\langle (jf_{\frac{7}{2}})_K|\tfrac{1}{2}V_{T=1}-\tfrac{1}{2}V_{T=0}|(p_{\frac{3}{2}}f_{\frac{7}{2}})_K\rangle .$$

As an example of a wave function resulting from the calculation we have, for the lowest $\frac{3}{2}$ state of ^{49}Sc,

$$\psi(\tfrac{3}{2}-) = 0.87\,(\tfrac{3}{2}\text{-s.p.}) + 0.24\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_1\right]_{\frac{3}{2}} +$$

$$+ 0.27\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_2\right]_{\frac{3}{2}} +$$

$$+ 0.34\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_3\right]_{\frac{3}{2}} + 0.084\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{5}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_1\right]_{\frac{3}{2}}$$

where the single-particle state of good isospin is given by

$$(\tfrac{3}{2}\text{-s.p.}) = \sqrt{\tfrac{8}{9}}\,\overset{p}{p}_{\frac{3}{2}} - \sqrt{\tfrac{1}{9}}\left[\overset{n}{p}_{\frac{3}{2}}\left(\overset{p}{f}_{\frac{7}{2}}\overset{n}{f}{}^{-1}_{\frac{7}{2}}\right)_0\right]_{\frac{3}{2}} .$$

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