LE JOURNAL DE PHYSIQUE

Classification
Physics Abstracts
24.70

Application of the semiclassical method to polarization

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(*Reçu le 16 août 1978, accepté le 4 octobre 1978*)

Résumé. — Une formulation de la polarisation en diffusion élastique est établie à partir de l'approximation semi-classique. La polarisation s'exprime comme une somme de deux termes, l'un proportionnel à l'angle et l'autre proportionnel à la dérivée de la section efficace. Les amplitudes relatives de ces deux termes dépendent de la phase entre le potentiel spin-orbite et le potentiel réel. Ce résultat est comparé à des calculs quantiques dans différents cas. Bien que quantitativement peu précise, cette formulation reproduit de manière qualitative dans diverses conditions le comportement général de la polarisation.

Abstract. — A formula is derived for the polarization in elastic scattering, based on the semiclassical approximation. The polarization is given by the sum of two terms, one proportional to the angle and one proportional to the derivative of the cross section. The relative magnitudes of the terms depend on the relative phase of the spin orbit potential with respect to the real potential. This is compared with quantum calculations for several cases. While not quantitatively accurate, the formula reproduces the qualitative behaviour of the polarization under a broad range of conditions.

1. **Introduction.** — The qualitative behaviour of polarization in elastic scattering has long been obscure [1]. In the 1950's, semiclassical expressions were derived for polarization based on the Thomas form for the spin-orbit interaction [2, 3]. This yields a polarization that is proportional to scattering angle θ, when the central potential is imaginary. While the experimental data often shows a smoothly rising polarization with angle, the polarization often displays oscillations resembling the derivative of the elastic scattering cross section. Models for this derivative behaviour have been constructed, but they are completely *ad hoc* [4, 5]. We shall present a description of polarization, based on the recently perfected version of semiclassical theory, which includes these two types of behaviour (linear with angle or oscillatory) as special cases.

In the modern semiclassical theory, scattering is described with a classical trajectory for the projectile together with a quantum phase for the particle on the trajectory. In cases of interest, there are two dominant trajectories leading from the incident beam direction to a given scattering angle. These trajectories pass each side of the nucleus, and their interference gives rise to the diffractive structure. Such a theory is well-suited to a simplified description of polarization phenomena, since the spin-dependent potential will differ along the two trajectories on opposite sides of the nucleus.

The remainder of this paper is organized as follows. In section 2 we recall the main results of the semiclassical method, together with some useful analytic approximations, and apply it to polarization. The main result is eqs. (2.20-2.21). In section 3 we compare the results with optical model calculations and experimental data. In section 4 we examine previous models, and summarize the results.

(*) Supported by the National Science Foundation under Grant No. PHY76-20097 A01.

2. **Semiclassical theory.** — The semiclassical theory using real trajectories is discussed by Ford and Wheeler [6]. An excellent review is given by Berry and Mount [7]. Quantum mechanical corrections have been obtained by using complex solutions of the classical equations of motion as shown by Miller [8] and Balian and Bloch [9]. In particular, an accurate description of diffraction is only possible when complex trajectories are included. This case was treated by Knoll and Schaeffer [10] and we shall use their notation and summarize their results.

The scattering amplitude f is written as a sum over trajectories j which lead to a scattering angle $\theta_j = \pm\,\theta$,

$$f(\theta) = \sum_j A_j(\theta)\, e^{iS_j(\theta_j)} . \tag{2.1}$$

Here S_j is the semiclassical action along trajectory j, given by

$$S_j = 2\int_{r_j}^{\infty} \left[\sqrt{2\,m(E - V(r)) - \left(\frac{b_j p}{r}\right)^2} - p\right] dr + \\ + b_j p(\pi - \theta_j) - 2 p r_j$$

with

$$\theta_j = \pi - 2\int_{r_j}^{\infty} \frac{p b_j}{\sqrt{[2\,m(E - V) - (b_j p/r)^2]}} \frac{dr}{r^2} . \tag{2.2}$$

This expression for S_j involves besides θ_j, the incident energy E, the scattering potential V, the incident momentum $p = (2\,mE)^{1/2}$, the impact parameter of the trajectory b_j, and the radial turning point of the trajectory, r_j. The amplitude A_j in eq. (2.1) is given by

$$A_j = \pm \sqrt{\frac{b_j}{\sin\theta \dfrac{d\theta}{db_j}}} . \tag{2.3}$$

In the nuclear surface, the potential is of the form

$$V = \frac{Z_1 Z_2 e^2}{r} - (V + iW) \exp\left[-\left(\frac{r - R}{a}\right)\right]$$

where R and a are the usual parameters of the more standard Woods-Saxon shape.

With this potential, it is possible to derive simple approximations for the deflection angle and the action [10]. In terms of the turning point r_0, the deflection angle is

$$\theta(r_0) = \frac{Z_1 Z_2 e^2}{r_0 E} - \left(\frac{V + iW}{E}\right) \times \\ \times \sqrt{\frac{\pi R}{2a}} \exp\left[-\left(\frac{r_0 - R}{a}\right)\right] . \tag{2.4}$$

We shall also need the Coulomb deflection separately, which is the first term of eq. (2.4), $\theta_c = Z_1 Z_2 e^2/r_0 E$. The action corresponding to the same trajectory with turning point r_0 is

$$S(r_0) = -p\frac{Z_1 Z_2 e^2}{E} \log\frac{R}{r_0} + pa\left(\frac{V + iW}{E}\right) \times \\ \times \sqrt{\frac{\pi R}{2a}} \exp\left[-\left(\frac{r_0 - R}{a}\right)\right] - p r_0 \theta . \tag{2.5}$$

In the usual diffraction scattering, there are two important trajectories. They can be obtained by solving (2.4) for r_0 with $\theta_j = \pm\,\theta$. We label these trajectories by (+) and (−). The turning point is complex and approximately given by

$$r^{\pm} \simeq \rho^{\pm} + i\left(\mp\, a\pi/2 - a \tan^{-1}\frac{V}{W}\right) \tag{2.6}$$

with the real part $\rho^{\pm}$ satisfying

$$\pm\,\theta = \frac{Z_1 Z_2 e^2}{E\rho^{\pm}} \pm \sqrt{\frac{V^2 + W^2}{E^2}} \times \\ \times \sqrt{\frac{\pi R}{2a}} \exp\left[-\left(\frac{\rho^{\pm} - R}{a}\right)\right] . \tag{2.7}$$

The action on these two trajectories is given by

$$S^{\pm} = \Sigma^{\pm} + iI^{\pm} \tag{2.8}$$

with

$$\Sigma^{\pm} = \frac{Z_1 Z_2 e^2}{E^2} \log \rho^{\pm}/R \mp pa \sqrt{\frac{\pi R}{2a}} \times \\ \times \sqrt{\frac{V^2 + W^2}{E^2}} \exp\left[-\left(\frac{\rho^{\pm} - R}{a}\right)\right] \mp p\rho^{\pm}\theta$$

and

$$I^{\pm} = pa\left[\pi/2 \pm \tan^{-1}\frac{V}{W}\right] \times \\ \times \sqrt{\frac{V^2 + W^2}{E^2}} \sqrt{\frac{\pi R}{2a}} \exp\left[-\left(\frac{\rho^{\pm} - R}{a}\right)\right] .$$

The cross section is given by

$$\sigma = \sigma^+ + \sigma^- - 2\cos(\Sigma^+ - \Sigma^-)\sqrt{\sigma^+\sigma^-} \tag{2.9}$$

where

$$\sigma^{\pm} = |A^{\pm}|^2 e^{-2I^{\pm}} .$$

We now consider the effect of a spin-orbit potential. The trajectories and action will differ for spin up and spin down. We assume this difference is small and write

$$\delta S^{\pm} = S^{\pm}_{\uparrow} - S^{\pm}_{\downarrow} . \tag{2.10}$$

We further assume that the polarization comes mainly from the variation in S, and not from the variation in $A^{(\pm)}$. Then the asymmetry in the cross section can be expressed

$$\delta\sigma = P\sigma \simeq -(\delta I^+ + \delta I^-)\,\sigma - (\delta I^+ - \delta I^-)(\sigma^+ - \sigma^-) + \\ + \delta(\Sigma^+ - \Sigma^-)\,2\sin(\Sigma^+ - \Sigma^-)\sqrt{(\sigma^+\sigma^-)} . \tag{2.11}$$

At forward angles, $\sigma^{+'} \simeq \sigma^-$ and the second term can be dropped. We rewrite the third term by noting that the variation in σ with angle is governed mainly by the interference in (2.9), i.e.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \simeq 2 \sin(\Sigma^+ - \Sigma^-) \frac{\mathrm{d}(\Sigma^+ - \Sigma^-)}{\mathrm{d}\theta} \sqrt{\sigma^+ \sigma^-} . \tag{2.12}$$

The derivative of the action with respect to the angle in eq. (2.12) can be replaced by the angular momentum :

$$\frac{\mathrm{d}(\Sigma^+ - \Sigma^-)}{\mathrm{d}\theta} = \mathrm{Re}\,(b^+ + b^-)\, p \simeq \mathrm{Re}\,(r_0^+ + r_0^-)\, p .$$

Thus the polarization becomes

$$P \simeq -(\delta I^+ + \delta I^-) - \frac{\delta(\Sigma^+ - \Sigma^-)}{\mathrm{Re}\,(r_0^+ + r_0^-)\, p} \frac{\mathrm{d}\sigma/\mathrm{d}\theta}{\sigma} . \tag{2.13}$$

If the spin-orbit potential is weak, the change in S for a given scattering angle is the same as the change in S holding the turning point fixed. This is because S is stationary along the trajectories and therefore, the variation of S with respect to r_0 are of second order in δr_0. The perturbative expression for the change in action is then

$$\delta S\,|_\theta = \delta S\,|_{r_0} = \int_{r_0}^{\infty} \frac{2 m V_{ls}(r)}{\sqrt{2 m[E - V(r)] - (bp/r)^2}} \mathrm{d}r . \tag{2.14}$$

With the approximation (2.5), the variation in S may be determined directly as

$$\delta S^\pm = k a_{LS} \sqrt{\frac{\pi R}{2 a}} \frac{\delta V}{E} \exp\left[-\left(\frac{r_0 - R}{a_{LS}}\right)\right]. \tag{2.15}$$

In principle one should find the solutions r_0 of (2.4) leading to a real scattering angle : $\theta(r_0) = \pm\,\theta$. These solutions are complex and produce the phase for $\delta S^\pm = \delta\Sigma^\pm + i\,\delta I^\pm$. A somewhat more transparent expression can be obtained by rewriting (2.15) as

$$\delta S^\pm = k a (\pm\,\theta - \theta_\mathrm{c}) \frac{\delta V \exp\left[-\left(\frac{r_0 - R_{LS}}{a_{LS}}\right)\right]}{(V + iW) \exp\left[-\left(\frac{r_0 - R}{a}\right)\right]} . \tag{2.16}$$

In case the geometries are different, the ratio in (2.16) depends on $r_0^\pm$, the solution of (2.4) for $\theta(r_0) = \pm\,\theta$. But if the central and spin orbit potentials have the same shape, the exponential factors disappear. This is an important approximation that will be discussed later. The change in the potential is

$$\delta V^\pm = \pm\, l_\pm \frac{\hbar^2}{(m_\pi c)^2} \frac{V_{s0}}{a r_0} = \pm\, l_\pm\, V_{LS} . \tag{2.17}$$

The angular momenta of the two trajectories will be nearly equal, $l_+ \simeq l_- \simeq l$, so the imaginary change in the action may be expressed (assuming V_{LS} real)

$$(\delta I^+ + \delta I^-) \simeq 2 k a \frac{W V_{LS}}{V^2 + W^2} l\theta . \tag{2.18}$$

For eq. (2.13), we also need the difference in the real parts of the action. The first term in eq. (2.16) nearly cancels in the difference, but the second term, depending on the Coulomb deflection, contributes with the same sign. We find

$$\delta\Sigma^+ - \delta\Sigma^- \simeq 2 k a \theta_\mathrm{c} \frac{-V V_{LS}\, l}{V^2 + W^2} . \tag{2.19}$$

The final formula for the polarization may then be written

$$P = \nu\theta + \mu \frac{\mathrm{d}\log\sigma}{\mathrm{d}\theta} \tag{2.20a}$$

with

$$\nu = 2 k a \frac{W V_{LS}}{V^2 + W^2} l$$

$$\mu = k a \frac{V V_{LS}}{V^2 + W^2} \theta_\mathrm{c} . \tag{2.20b}$$

In case the spin orbit and central potential are of different shapes, the $r_0^\pm$ dependence of the exponential factors in eq. (2.16) does not cancel out. The most important modification is then the appearance of an additional phase :

$$\mathrm{e}^{-i\alpha\, \mathrm{Im}\, r_0^\pm/a} , \qquad \alpha = \frac{a - a_{LS}}{a_{LS}}$$

in $\delta S^\pm$ as given by (2.16), so the association of ν with W and μ with V is modified. With the expression (2.6) for $\mathrm{Im}\, r_0^\pm$, one gets for small α

$$\nu = 2 k a_{LS} \frac{W V_{LS}}{V^2 + W^2} l \times \left[1 - \alpha \frac{V}{W} \tan^{-1}\left(\frac{V}{W}\right) + \alpha \frac{V}{W} \frac{\pi}{2} \frac{\theta_\mathrm{c}}{\theta}\right] \tag{2.21}$$

$$\mu = k a_{LS} \frac{V V_{LS}}{V^2 + W^2} \theta_\mathrm{c} \times \left[1 + \alpha \frac{W}{V} \tan^{-1}\left(\frac{V}{W}\right) - \alpha \frac{W}{V} \frac{\pi\theta}{2\,\theta_\mathrm{c}}\right].$$

Table I.

Reaction	Energy	V_R	r_R	a_R	W	W_{SF}	r_1	a_1	V_{s0}	r_{s0}	a_{s0}
p + ^{120}Sn ([a])	40 MeV	49.255	1.17	0.75	6.1	3.80	1.32	0.627	6.2	1.01	0.75
p + ^{200}Pb ([b])	1 GeV	− 8.7	1.126	0.627	59		1.123	0.659	0.56	1.126	0.584
Li + C ([c])	20 MeV	65.7	1.948	0.689	7.95		2.504	0.114	2.55	1.948	0.689

([a]) From reference [13].
([b]) This potential is from reference [15], except that the spin orbit strength has been adjusted to reproduce the data of reference [11].
([c]) From reference [14], with the spin orbit strength chosen to roughly reproduce the polarization.

In the case the spin-orbit potential contains an imaginary part, (2.20*b*) becomes ([1])

$$v = 2\,ka\,\frac{WV_{LS} - W_{LS}\,V}{V^2 + W^2}\,l$$
$$\mu = ka\,\frac{VV_{LS} + WW_{LS}}{V^2 + W^2}\,\theta_c\,. \qquad (2.22)$$

The first term in eq. (2.20) was given by Köhler using different arguments. A derivative term was also obtained by *ad hoc* arguments, but has never been displayed as in eqs. (2.20)-(2.22). The derivative term depends on the central potential having a component in phase with the spin-orbit potential, and also depends on there being a difference in shape between central and spin orbit. The important shape change can be either the result of the Coulomb part of the central potential, or the result of different radial dependences to the nuclear potential, as can be seen from the formula.

3. **Practical cases.** — We now apply the semi-classical theory to three situations where experimental measurements have been made, and compare with optical model calculations. The reactions and optical potentials used for the calculations are shown in table I.

The first reaction we consider is 40 MeV proton scattering. The optical potential we use is from the global fit of reference [13]. This potential predicts a cross section and polarization shown in figure 1. The polarization is obviously predominantly oscillatory with only a slight overall rise with angle. This is qualitatively what is expected from eq. (2.20) : the real potential is most important at low energies, and so the μ term should dominate the v term. Note also that the polarization is positive when the cross section increases, implying that μ is positive, as could be predicted by using (2.20). In figure 1 is indicated a fit to the polarization with the (v, μ) parameterization of eq. (2.20), shown as dots. The resulting v and μ coefficients are quoted in table II. We compare this with the prediction of eq. (2.20), using the potential strengths from table I in the formulas.

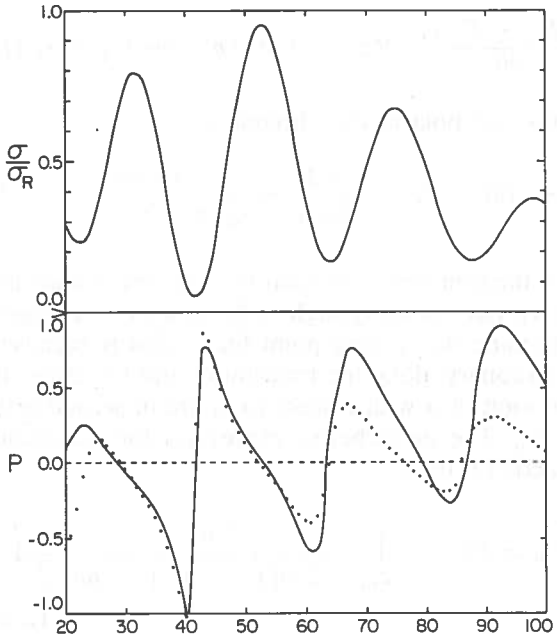

Fig. 1. — Proton scattering on ^{120}Sn at 40 MeV. The full line is the optical model calculation, and the dotted line is the fit to eq. (2.20*a*).

The result is shown in the second row of table II. The formula (2.20) which is extremely simple is seen to reproduce qualitatively the fitted v and μ coefficients. As a check, we have turned the spin-orbit potential into a purely imaginary one by multiplying it by a factor i. So, according to eq. (2.22), the $v\theta$ component of the polarization should dominate. From $v \sim -W_{LS}\,V$ the polarization should be negative. This is indeed verified in the full quantum calculation with the altered optical potential (Fig. 2). When the Coulomb interaction is turned off, drastic changes in the polarization can be observed, but the oscillatory behaviour does not fully disappear. So, all the physics of μ is not contained in the term proportional to θ_c. There are contributions from higher order terms that were neglected in (2.20).

We now turn to 1 GeV proton scattering on ^{208}Pb, measured in reference [11]. The experimental pola-

([1]) The factors V and W that multiply or divide α in the corrective factor of (2.21) are in this case to be replaced by $VV_{LS} + WW_{LS}$ and $WV_{LS} - W_{LS}\,V$ respectively. The term $\tan^{-1}(V/W)$ remains unchanged.

Table II. — *Coefficients v and μ appearing in* (2.20). *We first give the values v_f and μ_f fitted to the actual optical model calculation, as shown in the figures. The values v and μ are the approximate values obtained from* (2.20) *using simply the strengths of the optical potentials given in table I. For* 1 GeV *protons, the coefficient μ and μ depend on the scattering angle and have been evaluated at* $\theta = 10°$. *When the oscillatory term in the polarization is taken to be proportional to* $\frac{\theta \, d \log \sigma}{d\theta}$, *the coefficient is called μ'.*

	p + Sn	p + Pb	Li + C
v_f	0.1	2.5	0.3
μ_f	27×10^{-3}	-2×10^{-3}	7×10^{-3}
μ'_f		-12×10^{-3}	
v	0.34	2.8	0.05
μ	9×10^{-3}	-1×10^{-3}	5×10^{-3}
μ'		-6×10^{-3}	

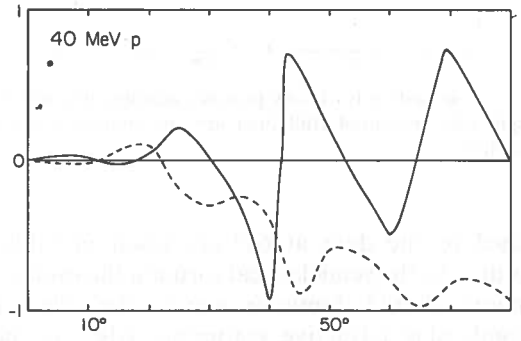

Fig. 2. — Comparison of the polarization obtained with a real spin orbit potential (full line) and an imaginary potential (dashed line), for proton scattering on ^{120}Sn at 40 MeV.

rization can be fitted with the optical potential of table I. With standard relativistic corrections ([2]), the cross section and polarization are shown in figure 2. The polarization rises smoothly until it saturates. Superimposed is a ripple proportional to $\frac{d \log \sigma}{d\theta}$. At 1 GeV, the central potential is predominantly imaginary so that v should indeed dominate. Also the real part is repulsive, so that μ is expected to be negative. Indeed, in figure 2 one can see the polarization has a maximum when the cross section is decreasing. In table II we give the values of v from eq. (2.20) together with the values extracted from figure 2. The agreement is satisfactory for v. The coefficient μ is small ($\mu \sim -1 \times 10^{-4}$) near $\theta \sim 10°$ and does not reproduce the fitted value. This is to be expected since the real part of the optical potential is extremely small, so higher order corrections cannot be neglected. They arise from the small difference in shape of V_{LS} and V, as shown in eq. (2.21). When $V\theta_c$ in the numerator of (2.20) is replaced by the more accurate form

$$V\theta_c - \alpha \frac{\pi}{2} W\theta , \qquad \alpha \simeq \frac{a_W - a_{LS}}{a_{LS}} , \tag{3.1}$$

the value obtained for μ is nearly correct as can be seen in table II. It is an order of magnitude larger, so Coulomb scattering plays no role at these energies for determining the value of μ.

From this correction factor, it can be seen that μ is actually proportional to θ. Indeed, the polarization can be best filled by

$$P = v\theta + \mu' \theta \frac{d \log \sigma}{d\theta} \tag{3.2}$$

as can be seen in figure 3. We have then

$$\mu' \sim -k(a_W - a_{LS}) \frac{\pi}{2} \frac{V_{LS} W}{V^2 + W^2} . \tag{3.3}$$

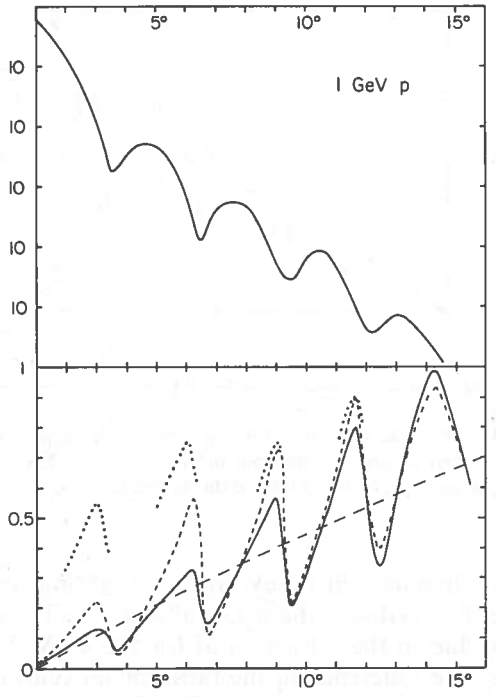

Fig. 3. — Proton scattering on ^{208}Pb at 1 GeV. Above : cross section ; below : polarization. The full line is the optical model calculation, the dashed line the fit using the parametrization of eq. (3.2). The dots indicate the values obtained with the parametrization (2.20*a*) when μ is adjusted to match the optical model calculations at $\theta = 10°$.

In table III, we compare the values of μ at 10° and of μ' obtained from a fit of optical model calculations to those calculated using (2.21) and (3.3). The agree-

([2]) Relativistic corrections can be handled easily [16]. Simply replace the actual energy E by $E(1 + (E/2m))$ and the potential V by $V(1 + (E/m))$.

Table III. — *Coefficients* μ_f *of* (2.20) *adjusted at* $\theta = 10°$ *to the optical model calculations done with various surface thickness parameters* a_{LS}. *The value obtained using* (2.21) *for* μ *is also given. The coefficients* μ_f' *and* μ' *are those of formula* (3.2), *the first being adjusted to the optical model calculations, the second calculated using* (3.3). *Note the sign change for* $a_{LS} \simeq a_W = 0.669$.

	$a_{LS} = 0.4$	0.6	0.8	1.2
μ_f	-7×10^{-3}	-3×10^{-3}	0×10^{-3}	$+5 \times 10^{-3}$
μ	-14×10^{-3}	-2×10^{-3}	$+4 \times 10^{-3}$	$+10 \times 10^{-3}$
μ_f'	-4×10^{-2}	-2×10^{-2}	0×10^{-2}	$+3 \times 10^{-2}$
μ'	-8×10^{-2}	-1×10^{-2}	$+2 \times 10^{-2}$	$+5 \times 10^{-2}$

ment is impressive, especially the change of sign in μ or μ' predicted by (2.21) and (3.3) is actually observed in the calculations (Fig. 4). The phase of the ripple is inverted whereas the cross section is barely affected by the variation of a_{LS}. The oscillations are out of phase with $d\sigma/d\theta$ for $a_W > a_{LS}$, for $a_W \simeq a_{LS}$ they do not follow the $d\sigma/d\theta$ rule, and for $a_W < a_{LS}$ the oscillations are again proportional to $d\sigma/d\theta$ and in phase.

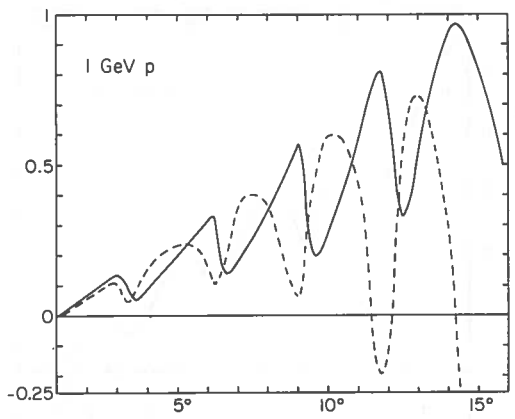

Fig. 4. — Polarization for 1 GeV protons with various surface thickness parameters for the spin orbit interaction. The full line corresponds to $a_{LS} = 0.584$ fm and the dashed line to $a_{LS} = 1.2$ fm.

To summarize, at 1 GeV, the polarization rises with angle. This is due to the strong absorption. The ripple is not due to the refraction as for the 40 MeV case but to the difference in the (absorptive) central and spin-orbit potential shapes. But the same ripples could be obtained by changing the ratio of real to imaginary part in the optical potential and using the same shapes. Microscopic calculations should show which explanation is the correct one.

Finally, we turn to a heavy ion reaction, $^6Li + ^{12}C$ measured at 20 MeV in reference [10]. In this case the optical model calculation does not fit the data very well. We have nevertheless used the parameters of reference [10] with a spin-orbit interaction strength adjusted so as to roughly reproduce the measured polarization. The result of the optical model calculation is shown in figure 5, together with a fit to the latter using eq. (2.20). The fitted parameters v_f and μ_f

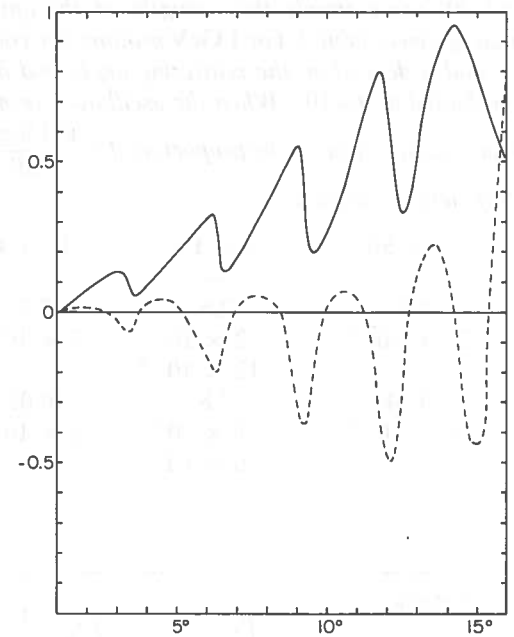

Fig. 5. — Polarization for 1 GeV proton scattering obtained with a real spin orbit potential (full line) and an imaginary potential (dashed line).

matched to the data at 63° are given in table II. According to the semiclassical formula the oscillatory behaviour should dominate due to the substantial Coulomb plus refractive scattering. Also the polarization should be positive ($\mu > 0$) at the places the cross section increases. This is indeed the case (Fig. 6).

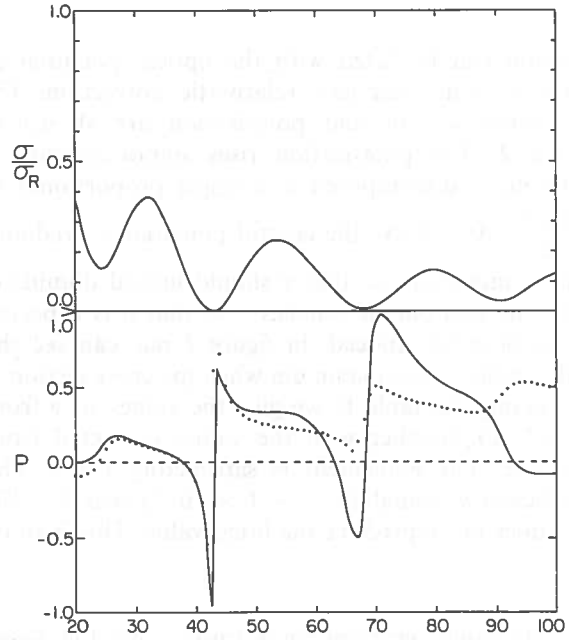

Fig. 6. — $^6Li + ^{12}C$ scattering at 20 MeV. Solid line is the optical model calculation ; dotted line is eq. (2.20*a*) fit at 40°.

In fact the μ coefficient calculated using (2.20) matches the fit to the optical model calculations. There is, however, some indication of a $v\theta$ component both in the optical model calculation and in the data. The semiclassical estimate of v ($\sim$ 0.05) using (2.20) is however much lower than the fitted values ($\sim$ 0.3). This is to be related to the crudeness of (2.20) when the shapes are so different ($a_V = a_{LS} = 0.689$ but $a_W = 0.114$). Taking the value of the potentials at $r = 5.73$ fm which is the radius of the imaginary part instead of just the strength in (2.20) leads to $v \sim 0.2$ which is the correct order of magnitude. It can also be checked in this case that the use of an imaginary spin-orbit potential leads to $P \sim v\theta$ with $v < 0$ in both (2.22) and the optical model calculations. So, indeed (2.20)-(2.12) reproduce the qualitative features of the Li + C polarization. The latter can be seen to be of the oscillatory type, as for 40 MeV proton scattering on Sn. In both cases, the explanation for the predominantly oscillatory polarization was the presence of a strongly refractive nuclear potential.

4. **Summary and discussion.** — The semiclassical model separates the polarization into two main categories, depending on the relative phase of the spin orbit potential and the real potential.

This is summarized by the two coefficients v and μ we have introduced. The semiclassical theory gives values for v and μ which are in qualitative agreement with those filled to actual calculations. Especially the sign of v and μ can be obtained using the approximate formula (2.20). The $v\theta$ term in the polarization dominates when the imaginary part of the scattering potential is large (assuming a real spin-orbit potential). For a positive spin-orbit potential (this is the usual sign), and an absorptive central potential, v is positive. The derivative term shows that the polarization oscillates whenever the nuclear attraction is strong enough. Its existence depends on the occurrence of Coulomb scattering. For attractive potentials, we have $\mu > 0$ (assuming the spin-orbit term has its standard sign) which implies that the polarization is in phase with $d\sigma/d\theta$. This turns out to be the case for p + Sn at 40 MeV as well as Li + C at 20 MeV. For spin-orbit potentials with a sharp edge and strongly absorptive central potentials, μ is negative. This is the case for p + Pb at 1 GeV. This qualitative change appears both in the fully quantum mechanical calculations and in the semiclassical formula (2.20)-(2.21).

These expressions are quite different from the one given by Rodberg [4], namely $\mu \sim kR \frac{V_{LS}}{E}$ which does not predict any sign change.

A simple error in the use of the semiclassical theory gives rise to this expression, which we now show. Combining eq. (2.15) and (2.17) yields

$$\delta S^{\pm} = \pm\, l_{\pm}\, ka\gamma \frac{V_{LS}}{E} \exp\left[-\left(\frac{r_0^{\pm} - R}{a}\right)\right], \qquad \gamma = \sqrt{\frac{\pi R}{2a}}. \tag{4.1}$$

From (2.6) we see that the imaginary part of $r_0^{\pm}$ varies sufficiently so as to change the sign of the exponential, hence

$$\delta S^{\pm} \simeq l_{\pm}\, ka\gamma \frac{V_{LS}}{E} \exp\left[-\left(\frac{\rho_0^{\pm} - R}{a}\right)\right] \tag{4.2}$$

with $\rho^{\pm}$ real. The more usual WKB evaluation of the classical action for two trajectories passing on either side of the nucleus at the same distance r (assumed to be real) leads to

$$\delta S^{\pm} = \pm\, l_{\pm}\, ka\gamma \frac{V_{LS}}{E} \exp\left[-\left(\frac{r - R}{a}\right)\right]. \tag{4.3}$$

One thus obtains

$$\mu = ka\sigma \frac{V_{LS}}{E} \exp\left[-\left(\frac{r - R}{a}\right)\right] = k \sqrt{\frac{\pi}{2} ra}\, \frac{V_{LS}}{E} \exp\left[-\left(\frac{r - R}{a}\right)\right], \tag{4.4}$$

which is essentially the result of Rodberg [4]. This is however incorrect since the two trajectories considered in (4.3) do not correspond to a stationary classical action. The correct result is (4.1) or (4.2) which differs from (4.3) by a ($\pm$) sign. When (4.2) is used, there is almost a perfect cancellation between $\delta\Sigma^+$ and $\delta\Sigma^-$. This explains why analytic expressions of μ relying on the calculation of Σ^+ and Σ^- are rather delicate to obtain.

A non-zero value of μ can be obtained by considering the difference between ρ^+ and ρ^-, which is the origin of the term proportional to θ_c. Another way to get a non-zero value of μ is to use different shapes for the spin-orbit and the central Woods-Saxon potential. Then in eq. (4.1) there is an additional phase which is different for positive and negative angle scattering, preventing the total cancellation in Re $\delta(S^+ - S^-)$.

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