Threshold pion absorption in nuclei*

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We attempt to explain threshold pion absorption in nuclei using simple pion rescattering mechanicsms. While the model is successful in the case of the deuteron it leads to 10–30% too small absorption rates in other nuclei. We also study a $\rho\omega$ exchange absorption mechanism that does not act in the deuteron, but find that it can only remove a small part of the remaining discrepancy. The two-nucleon absorption mechanism accounts for 75% of the imaginary part of the S-wave pion nucleus optical potential.

[NUCLEAR REACTIONS Pion absorption calculated at threshold in light nuclei ^{2}H, ^{4}He, ^{12}C, ^{16}O, ^{20}Ne, and nuclear matter.]

I. INTRODUCTION

Pion absorption in nuclei has been discussed extensively in recent literature.[1] The lack of success in early attempts to explain threshold pion absorption with single nucleon mechanisms has led to a general consensus that the absorption mechanism involves at least two nucleons. Even so it was not long ago still claimed that "by and large, pion absorption in nuclei is still a mystery."[2] The recent progress in calculations of pion absorption in the deuteron using simple two-body rescattering mechanisms[3-5] suggests that it is now high time to reduce some of the "mystery" surrounding pion absorption in nuclei.

In this paper we attempt to explain pion absorption at threshold with the assumption that the dominant mechanism is absorption of a virtual pion, created in a scattering of the initial physical pion. To describe the rescattering vertex we use the phenomenological zero range $\pi\pi NN$ interaction Hamiltonian employed by Woodruff[6] and later by Koltun and Reitan.[7] This interaction involves two coupling parameters which are determined by the pion nucleon S-wave scattering lengths.

With this model we obtain perfect agreement with the empirical absorption rate for the deuteron, but find that the calculated absorption rates in other nuclei are 10–30% too small. These results represent an improvement over the earlier situation where the predicted absorption rate in ^{4}He was found to be too large by about 50% (Refs. 8 and 9) and in nuclear matter too small by a factor 3.[10] To understand the remaining discrepancy in nuclei other than the deuteron we also consider a two-body process involving exchange of a ρ and a ω meson which does not act in the deuteron. We find, however, that this process can increase the calculated absorption rate by at most 5% and that it is thus insufficient to remove all of the discrepancy.

It was recently suggested that the off-shell pion-nucleon scattering amplitude should differ considerably from the on-shell one.[11] The smooth off-shell behavior of the pion rescattering amplitude we employ and the subsequent fair (perfect in the deuteron case) agreement with the empirical absorption rates in general indicates that no drastic off-shell extrapolation of the amplitudes is required.

The present paper falls into six sections. In Sec. II we explain the pion rescattering model and the method of calculating the imaginary parts of the S-wave scattering lengths. In Sec. III we give the numerical results for pion rescattering in light nuclei and study the effect of wave function correlations. In Sec. IV we calculate the imaginary part of the S-wave pion-nucleus optical potential using the Fermi gas model. In Sec. V we develop the absorption operator involving pion absorption into a ρ and an ω meson and calculate its magnitude. Finally, we give a concluding discussion in Sec. VI.

II. ABSORPTION RATE FOR PION RESCATTERING

A. General expressions for the absorption rate

The rate for pion absorption in a nucleus is

$$\Gamma = 2\pi \sum_{if} \delta(E_f - E_i - \omega) \left| T_{if} \right|^2 , \tag{2.1}$$

where T_{if} is the matrix element of the absorption operator and the sum runs over all final states. The energies of the nuclear ground state and final

states are denoted E_i and E_f, and the pion energy ω.

The π absorption Hamiltonian will be a two particle operator in the nucleon space. We will basically deal with the independent particle model for the nucleon wave functions, modified to take into account short range correlations. In the independent particle model, the integral over nucleons can be written in the form

$$\Gamma = 2\pi \sum_{\substack{k<l \\ m<n}} \left| \int d(1)d(2)\phi_m^*(1)\phi_n^*(2)T(12)[\phi_k(1)\phi_l(2) - \phi_k(2)\phi_l(1)] \right|^2 \delta(E_{mn} - E_{kl} - \omega) . \tag{2.2}$$

Here (1) and (2) represent all the coordinates of nucleons 1 and 2. The operator $T(12)$ is the two-body pion absorption operator, to be derived in the next section. The sums over orbit labels k, l range independently over all occupied orbits, and the sum m, n over vacant orbits. The energy of the pair of wave functions kl is denoted E_{kl} and that of the pair mn E_{mn}.

A simpler equivalent representation is obtained with the use of normalized antisymmetric pair wave functions:

$$\psi_{kl}(12) = \frac{1}{\sqrt{2}} [\phi_k(1)\phi_l(2) - \phi_k(2)\phi_l(1)] . \tag{2.3}$$

Equation (2.2) then becomes

$$\Gamma = 2\pi \sum_{i,f} \left| \int d(1)d(2)\psi_i(12)T(12)\psi_f(12) \right|^2 \times \delta(E_f - E_i - \omega) , \tag{2.4}$$

where now the labels i, f indicate occupied and vacant antisymmetric pair states. The corresponding energies are denoted E_i and E_f.

B. Pion absorption operator

We assume that S-wave absorption takes place after one rescattering as illustrated in Fig. 1. To describe the rescattering vertex we use the phenomenological Hamiltonian for the $\pi\pi NN$ interaction suggested by Woodruff[6] and used also by Koltun and Reitan[7]:

$$H = 4\pi \frac{\lambda_1}{\mu} \bar{\psi} \vec{\phi} \cdot \vec{\phi} \psi + 4\pi \frac{\lambda_2}{\mu^2} \bar{\psi} \vec{\tau} \cdot \vec{\phi} \times \vec{\pi} \psi . \tag{2.5}$$

Here ψ and $\bar{\psi}$ are the nucleon field operators and $\vec{\phi}$ the isovector pion field operator. The momentum operator conjugate to the pion field operator is denoted $\vec{\pi}$. In (2.5) $\vec{\tau}$ is the nucleon isospin operator and μ the pion mass ($\mu = 0.7$ fm$^{-1} = 139$ MeV). The two coupling constants λ are determined by the S-wave pion-nucleon (πN) scattering lengths as

$$\lambda_1 = -\tfrac{1}{6} \mu(1 + \mu/m)(a_1 + 2a_3) ,$$
$$\lambda_2 = \tfrac{1}{6} \mu(1 + \mu/m)(a_1 - a_3) . \tag{2.6}$$

Here m is the nucleon mass. With the numerical values for the scattering lengths of Bugg, Carter, and Carter[12] one obtains $\lambda_1 = 0.003 \pm 0.001$ and $\lambda_2 = 0.05 \pm 0.001$. With these values the Hamiltonian (2.7) leads to an adequate fit to the S-wave πN phase shifts up to 50 MeV pion laboratory kinetic energy.[5]

To construct the expression for the two-body rescattering operator one also needs the interaction Hamiltonian for the final pion absorption vertex:

$$H = -\frac{f}{\mu} \bar{\psi} (\vec{\sigma} \cdot \vec{\nabla}) \vec{\phi} \cdot \vec{\tau} \psi . \tag{2.7}$$

Here $\vec{\sigma}$ is the nucleon spin operator and $f^2/4\pi = 0.08$. Using (2.5) and (2.7) one finds the expression for the S-wave rescattering operator corresponding to the diagram in Fig. 1 to be

$$T = -i \frac{8\pi\sqrt{2}}{\mu(2\omega_{\vec{q}})^{1/2}} f \frac{\sigma^2 \vec{k}}{\mu^2 + \vec{k}^2 - \omega_{\vec{k}}^2} \times \left(\frac{\lambda_1}{\mu} \tau_+^2 - i \frac{\lambda_2}{2\mu^2} (\omega_{\vec{q}} + \omega_{\vec{k}})(\tau^1 \times \tau^2)_+ \right) . \tag{2.8}$$

Here $\vec{q}$ is the momentum and $\omega_{\vec{q}}$ the energy of the initial pion and $\vec{k}$ and $\omega_{\vec{k}}$ the momentum and energy of the rescattered pion. Near threshold it is natural to assume that the energy of the pion is equally shared between the nucleons, in which case $\omega_{\vec{k}} = \frac{1}{2}\omega_{\vec{q}} \simeq \frac{1}{2}\mu$. With this value for the energy transfer

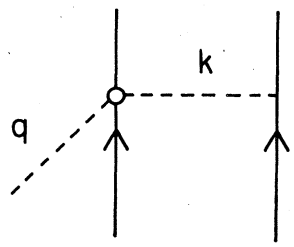

FIG. 1. Two-nucleon absorption mechanism involving pion rescattering.

the effective pion mass in the exchange operator is $\mu^* = (\mu^2 - \omega_{\vec{k}}^2)^{1/2} = \sqrt{3}\,\frac{1}{2}\mu$.

To obtain the properly symmetrized rescattering operator for the nucleon pair (12) we add to (2.8) the operator corresponding to the Feynman diagram in which the pion rescatters off nucleon 2 rather than nucleon 1. By taking the Fourier transform of the combined operator and letting $\vec{q} \to 0$ we finally obtain the threshold S-wave absorption operator in the form

$$T(12) = t^1(\tau^1 + \tau^2)_+ + t^2(\tau^1 - \tau^2)_+ - it^3(\tau^1 \times \tau^2)_+, \qquad (2.9)$$

with the amplitudes t^m given as

$$t^1 = \frac{f}{\mu^{5/2}} \lambda_1 \mu^{*2} \overline{Y}_1(\mu^* r)(\vec{\sigma}^1 - \vec{\sigma}^2)\cdot\hat{r}, \qquad (2.10a)$$

$$t^2 = \frac{f}{\mu^{5/2}} \lambda_1 \mu^{*2} \overline{Y}_1(\mu^* r)(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r}, \qquad (2.10b)$$

$$t^3 = -\frac{3f}{2\mu^{5/2}} \lambda_2 \mu^{*2} \overline{Y}_1(\mu^* r)(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r}. \qquad (2.10c)$$

Here the function $\overline{Y}_1$ is defined to be

$$\overline{Y}_1(x) = \left(1 + \frac{1}{x}\right)\frac{e^{-x}}{x}. \qquad (2.11)$$

In (2.10) the vector $\vec{r}$ is defined as $(\vec{r}_1 - \vec{r}_2)$, with $\vec{r}_1$ and $\vec{r}_2$ being the coordinates of nucleons 1 and 2. We now turn to the evaluation of the matrix elements of this operator.

C. Spin and isospin matrix elements

It is convenient to first calculate the spin and isospin matrix elements of the operator (2.9) before dealing with the radial part. The operator t^1 acts only between isospin 1 pairs, and flips the spin from 0 to 1 or vice versa. The operators t^2 and t^3 act only between spin 1 pairs, and flip the isospin. Making the isospin of the ground state pair explicit in the sum (2.4) and evaluating the sums over initial and final isospin components leaves the square of the transition matrix element in the form

$$|T_{if}|^2 = \left(2\sum^{T=0} |t^2_{if} - t^3_{if}|^2 + \sum^{T=1} \left(\tfrac{4}{3}|t^1_{if}|^2 + \tfrac{2}{3}|t^2_{if} + t^3_{if}|^2\right)\right) \qquad (2.12)$$

if the ground state has $T_z = 0$. The sums in (2.12) are over all these pairs with given T value (0 or 1).

Let us furthermore explicitly separate the spin dependence of the operators as

$$t^1 = (\sigma^1 - \sigma^2)\cdot\vec{V}^1, \qquad t^{2,3} = (\sigma^1 + \sigma^2)\cdot\vec{V}^{2,3}. \qquad (2.13)$$

The transition matrix element may be then decomposed into a sum over the different spin combinations of the pairs, by performing the spin sums in the matrix elements. The remaining matrix element is

$$|T_{if}|^2 = 16\left(\frac{1}{9}\sum^{11}(|\vec{V}^1_{if}|^2 + |\vec{V}^2_{if} + \vec{V}^3_{if}|^2) + \frac{1}{3}\sum^{01}|\vec{V}^1_{if}|^2 + \frac{1}{3}\sum^{10}|\vec{V}^2_{if} - \vec{V}^3_{if}|^2\right). \qquad (2.14)$$

Here the sums are taken over those pairs with given values S and T in the ground state wave function. The matrix element of the vector operator $\vec{V}_{if}$ is given by the integral over the spatial wave functions of the pairs,

$$\vec{V}^m_{if} = k^m \int \psi_i(\vec{r}_1, \vec{r}_2)\hat{r}\overline{Y}_1(\mu^* r_{12})\psi_f(\vec{r}_1, \vec{r}_2)d^3r_1 d^3r_2, \qquad (2.15)$$

with

$$k^1 = k^2 = f\frac{\mu^{*2}}{\mu^{5/2}}\lambda_1, \quad k^3 = -\tfrac{3}{2}f\frac{\mu^{*2}}{\mu^{5/2}}\lambda_2.$$

At this point we will expand the pair wave function in terms of c.m. and relative motion components. For the initial pairs, this will in general give some distribution over normalized relative wave functions,

$$\psi_i(\vec{r}_1, \vec{r}_2) = \sum a^i_{k_i, nlm}\psi_{k_i}(\vec{R})\phi_{nlm}(\vec{r}_{12}), \qquad (2.16)$$

with

$$\sum_{k_i, nlm} |a^i_{k_i, nlm}|^2 = 1.$$

For the final state, we shall make the assumption that the important final states form a complete set, in which case we may write

$$\phi_f(\vec{r}_1, \vec{r}_2) = \psi_{k_f}(\vec{R})\phi_{\bar{n}\bar{l}\bar{m}}(\vec{r}_{12}), \qquad (2.17)$$

with $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ and where $\bar{n}$, $\bar{l}$, $\bar{m}$ and k_f run over all values. This is of course only strictly valid for the deuteron. In the heavier nuclei the exclusion principle can limit the density of final states and furthermore the Hamiltonian of the final state could couple the c.m. and relative wave functions. We feel that these effects are insignificant, and that the approximation (2.17) should be adequate. The separation (2.17) allows the c.m. integral to be performed and we are left with

$$\vec{V}^\alpha_{if} = k^\alpha a^i_{k_f, nlm}\int d^3r\,\phi^*_{nl}(r)Y^{l*}_m(\hat{r})\hat{r}\overline{Y}_1(\mu^* r)Y^{\bar{l}}_{\bar{m}}(\hat{r})\phi_{\bar{n}\bar{l}}(r). \qquad (2.18)$$

The angular integration can be carried out to give a product of 3-J symbols. When this matrix element is squared and summed over final m states, the expression reduces to

$$\vec{\mathrm{V}}_{if}^{\alpha\dagger}\cdot\vec{\mathrm{V}}_{if}^{\beta}=k^{\alpha}k^{\beta}\sum_{f}(a_{f,nlm}^{ST})^2(2\bar{l}+1)\begin{pmatrix} l & \bar{l} & 1\\ 0 & 0 & 0\end{pmatrix}^2 I_{nl\bar{n}\bar{l}}^2\,, \tag{2.19}$$

where $I_{nl\bar{n}\bar{l}}$ is the radial integral

$$I_{nl\bar{n}\bar{l}}=\int r^2 dr\phi_{nl}(r)\bar{Y}_1(\mu^* r)\psi_{\bar{n}\bar{l}}(r)\,. \tag{2.20}$$

It is also convenient to define a pair probability B_{nl}^{ST} for pairs with quantum numbers $nlST$, summing over the c.m. states,

$$B_{nl}^{ST}=\sum_{i}|a_{k_i nl}^{ST}|^2\,. \tag{2.21}$$

Combining Eqs. (2.1), (2.15), (2.20), and (2.22) we arrive at the final expression for the transition rate,

$$\begin{aligned}\Gamma=\frac{32\pi}{3}f^2\frac{(\mu^*)^4}{\mu^5}\sum_{f}\delta(E_f-E_i-\omega)\\ \times\sum_{n}\left[B_{nl}^{01}\lambda_1^{\,2}+B_{nl}^{10}(\lambda_1+\tfrac{3}{2}\lambda_2)^2\right.\\ \left.+\tfrac{1}{3}B_{nl}^{11}(\lambda_1^{\,2}+(\lambda_1-\tfrac{3}{2}\lambda_2)^2\right]\\ \times(2\bar{l}+1)\begin{pmatrix} l & \bar{l} & 1\\ 0 & 0 & 0\end{pmatrix}^2 I_{nl\bar{n}\bar{l}}^2\,.\end{aligned} \tag{2.22}$$

To proceed from this point the explicit form for the pair wave functions and coefficients B must be provided. In the next section we construct pair wave functions for the deuteron and other light nuclei. It is convenient to express the derived absorption rate in terms of the imaginary part of the pion-nucleus scattering length. These quantities are related by

$$\mathrm{Im}a=\frac{\mu\Gamma}{4\pi}\,. \tag{2.23}$$

In Sec. IV below, we use the pair wave functions of infinite nuclear matter to calculate the imaginary part of the S-wave pion-nucleus optical potential. This is conventionally expressed in terms of a scattering length parameter B_0 (Ref. 10)

$$\frac{\Gamma}{\Omega}\simeq -2\,\mathrm{Im}U_{\mathrm{opt}}=\frac{4\pi}{\mu}\,\mathrm{Im}B_0\rho^2\,, \tag{2.24}$$

where ρ is the nucleon density and Ω the nuclear volume.

III. APPLICATION TO LIGHT NUCLEI

A. Overview

The Eq. (2.22) looks rather complicated, but in practice only a few terms are significant. First of all, the scattering length λ_1 is close to zero, so only the $(ST)=(10)$ and (11) pairs contribute significantly to pion absorption. The deuteron has only a single (10) pair, and the calculated deuteron absorption rate therefore provides a good check on this term in the transition rate expression. The next nucleus considered, ^{4}He also does not have any (11) pairs, and so ought to provide a test of whether in fact the absorption on (01) pairs is negligible. Heavier nuclei have (11) pairs, but due to the spatial antisymmetry of these pair wave functions, the associated radial matrix elements will be small. Thus our results will depend almost completely on the $(ST)=(10)$ pairs.

B. Pair wave functions

For the deuteron, no approximation is needed for the pair wave function, once the nucleon-nucleon potential is specified. We shall use the soft-core Reid potential and corresponding deuteron wave function.[13] For the other light nuclei, the simplest model is the independent particle model with harmonic oscillator wave functions, since the pair wave functions are then also oscillator functions. If the single particle wave functions fall off as $e^{-\nu r^2/2}$, the pair wave functions fall off as $e^{-\nu r_{12}^{\,2}/4}$. The oscillator size parameter ν is at our disposal, and we shall choose it to fit the empirically determined radii of the nuclei. The rms radii given in Ref. 14, corrected for the finite size of the proton, are given in Table I, together with the derived oscillator frequencies ω_{osc} and size parameter ν.

The short-range correlations could be important in making accurate estimates of the matrix elements of short-range operators of the type that we are considering. We shall therefore in addition to the oscillator model use Brueckner theory wave functions. This is discussed in Sec. IIID; in the remainder of this section we consider the integrals with pure oscillator functions.

For the final state it should be immaterial whether a plane wave or oscillator basis is used as the important quantity is the density of states $\sum_f\delta(E_f-E_0-\omega)|\phi_f(r)|^2$ at an energy $E_f\sim 140$ MeV and distances r in the range of 1 fm. We shall use an oscillator basis with $\omega_f=11$ MeV. Then the density of states is $|\phi_{nl}(r)|^2(1/2\omega_f)$. We shall

TABLE I. Root-mean-square radii (Ref. 14) and equivalent oscillator parameters for light nuclei.

Nucleus	$(\langle r^2\rangle_{\text{point}})^{1/2}$ (fm)	ω_{osc} (MeV)	ν (fm^{-2})
^{4}He	1.51	20.5	0.493
^{12}C	2.32	15.7	0.379
^{16}O	2.60	13.2	0.319
^{20}Ne	2.91	11.9	0.286

TABLE III. S-wave radial integrals [defined in Eq. (2.20)]. The dependence on ω_i with ω_f = 11 MeV.

$I_{nl\bar{n}\bar{l}}$	nl =	00	10	20
ω_i	$\bar{n}\bar{l}$ =	51	61	71
20.5		0.353		
15.7		0.298	0.325	
13.2		0.265	0.295	
11.9		0.246	0.276	0.287

evaluate the sum over final states in this fashion, using a single n_f and replacing the δ function by $1/2\omega_f$,

$$\delta(E_f - E_0 - \omega) \simeq \frac{\delta_{\bar{n}_s n}}{2\omega_f}\,,$$
$$(2n + \bar{l} + \tfrac{3}{2})\omega_f = E_f - E_0 - \omega\,. \tag{3.1}$$

In the case of absorption from $nl = (00)$, the nearest oscillator final state is $n = 5$, when $\omega_f = 11$ MeV.

We now turn to a discussion of the radial integrals, Eq. (2.22). A number of integrals, including all those needed for the ^{12}C calculation, are listed in Table II. First one should note that the S to P matrix element is 3 times larger than the next largest matrix element. As the transition rate depends on the square of the radial integrals (except for the tensor force mixing, considered in the next section) it is clear that only the $S \to P$ pair transition is important for calculation of the rate to an accuracy of ~10%. This point was mentioned earlier; to a reasonable approximation only (ST) = (100) pairs contribute.

The second observation to be made from Table II is that matrix elements are rather independent of the quantum number n of the final state. As a consequence, the choice of basis for the final state will not be at all critical, and also the approximation (3.1) for the density of states should be quite adequate.

We next examine the dependence of the matrix elements on ω_i. All the S to P matrix elements needed in our calculation are listed in Table III. The matrix elements decrease with ω_i, scaling approximately as $\omega^{2/3}$ power. This is close to the scaling $\sim\omega^{3/4}$ which is expected for a short-range operator. In fact, the operator $Y_1(\mu^* r)$ has a finite range and the peaks of the integrands in (2.20) occur near $r = 0.8$ fm. Since the absorption rate varies as the square of the integrals, the uncertainty in the empirical rms radii of a percent or so leads to a five percent uncertainty in the predictions based on the oscillator model.

TABLE II. Radial integrals with oscillator functions defined in Eq. (2.20). The parameters are ω_i = 15.7 MeV, ω_f = 11 MeV, and $\mu^* = (3/4\mu)^{1/2} = 0.606$ fm^{-1}.

$I_{nl\bar{n}\bar{l}}$	nl =	00	10	02	01	01
	$\bar{l}$ =	1	1	1	0	2
$\bar{n} = 5$		0.298	0.321	0.027	0.087	0.091
$\bar{n} = 6$		0.294	0.325	0.022	0.076	0.087
$\bar{n} = 7$		0.291	0.327	0.019	0.067	0.083

C. Oscillator model applied to light nuclei

We still need to determine the coefficients B_{nl}^{ST} in Eq. (2.22). For nuclei more complex than the deuteron, we shall assume maximum spin-isospin symmetry, in which case the spin and isospin decomposition is related to the total number of pairs with given nl by

$$B_{nl}^{10} = B_{nl}^{01} = \tfrac{1}{2}B_{nl}, \quad l \text{ even},$$
$$B_{nl}^{11} = 9B_{nl}^{00} = \tfrac{9}{10}B_{nl}, \quad l \text{ odd}. \tag{3.2}$$

For the α particle, there are six pairs, three with (ST) equal (10) and three with (01). For heavier nuclei, the relative coordinate decomposition may be done either explicitly with harmonic oscillator operator algebra or by using a table of transformation brackets. For the open shell nuclei ^{12}C and ^{20}Ne a further specification of the wave function is needed; we use the maximally deformed states of the shell model space. The results for the coefficients B_{nl} are given in Table IV. Note the the total number of pairs is just $\frac{1}{2}A(A-1)$.

We now evaluate the scattering lengths, using the oscillator integrals from section (3.2), and the pair coefficients from Table IV. Combining Eqs. (2.23) and (2.24) we obtain the scattering length results given in Table V. We see that the model falls short of explaining the data by a substantial amount in ^{4}He. In the heavier nuclei, agreement is better. The experimental points follow a linear dependence in A except that ^{12}C is somewhat high. The theory also has a smooth increase with A which arises from the rapid increase in the number of pairs and the smooth decrease of the radial integrals with A. As compared to a linear dependence on A, the ^{12}C theoretical rate is relatively

TABLE IV. Decomposition of pair of wave functions into relative coordinate oscillator functions.

	^{4}He	^{12}C	^{16}O	^{20}Ne
B_{00}	6	27	42	54.75
B_{10}		2	3	6
B_{20}				0.45
B_{02}		7	15	27
B_{12}				1.28
B_{04}				0.52
B_{01}		30	60	87.5
B_{11}				5.5
B_{13}				7
Total	6	66	120	190

enhanced, in qualitative agreement with the experimental systematics. The fact that ^{4}He is underpredicted would suggest that the $^1S_0$ pairs play a more important role than given to them in the simple rescattering model. The fact that the heavier nuclei are in better agreement with experiments would suggest that the spatially antisymmetric pairs indeed play the small role given to them in the model and that there is not too much room for modifying the $^1S_0$ absorption.

D. Tensor correlations and deuteron result

The Eqs. (2.14) and (2.22) were derived assuming that the spin sum could be carried out independently of the sum over m states of the spatial wave functions. This assumption is violated by the short-range tensor interaction between particles. If we consider pairs with l and S coupled to definite J, the pair wave function is

$$\psi(12)=\sum_{\substack{NLM\\ nlm}} a_{NLnlJ}\psi_{NLM}(R)\phi^J_{nl}(r)(Y^lS)^J_m . \tag{3.3}$$

The sums over M and m now do not lead to incoherence between terms with $l\neq l'$ in the squared matrix element. In the case of the coupled ^3S-^3D wave function, the matrix element becomes

TABLE V. The absorptive pion scattering lengths Ima for light nuclei in the oscillator model. The units are μ^{-1}.

Nucleus	Theory (μ^{-1})	Experiment (Ref. 2) (μ^{-1})
^{4}He	0.020	0.030
^{12}C	0.072	0.094
^{16}O	0.091	0.109
^{20}Ne	0.112	0.128

$$|T_{if}|^2=\tfrac{16}{3}(k^2-k^3)^2\sum_{NL}\Big|\sum_n\Big(a_{NL,n01}I_{n0,\bar{n}\bar{l}}+\frac{1}{\sqrt{2}}a_{NL,n21}I_{n2,\bar{n}\bar{l}}\Big)\Big|^2 . \tag{3.4}$$

For the deuteron there is, of course, no sum over c.m. states and the result may be expressed as

$$\Gamma=\frac{32\pi}{3}f^2\frac{(\mu^*)^4}{\mu^{5/2}}(\lambda_1+\tfrac{3}{2}\lambda_2)^2\sum_{\bar{n}\bar{l}}\Big|I_{00,\bar{n}\bar{l}}+\frac{1}{\sqrt{2}}I_{02n\bar{l}}\Big|^2 \times\delta(E_f-E_0-\omega) , \tag{3.5}$$

with the integrals defined using the deuteron wave functions $u(r)$ and $w(r)$

$$I_{00,\bar{n}\bar{l}}=\int r\,dr\,u(r)\bar{Y}_1(\mu^*r)\phi_{\bar{n}\bar{l}}(r) ,$$

$$I_{02,\bar{n}\bar{l}}=\int r\,dr\,w(r)\bar{Y}_1(\mu^*r)\phi_{\bar{n}\bar{l}}(r) . \tag{3.6}$$

Equation (3.5) is equivalent to an expression for the deuteron absorption length given by Brack, Riska, and Weise.[5] In this earlier work, μ^* was assumed equal to μ, and consequently the result was different. Using the Reid wave function in Eq. (3.6), and an oscillator final state with $\omega_f=11$ MeV we obtain $I_{00,51}=0.204$ and $I_{02,51}=0.074$. Combining Eq. (2.24) and (3.5), we find for the deuteron scattering length the value $\mathrm{Im}a_d=3.7\times10^{-3}\ \mu^{-1}$ in good agreement with the experimental value $0.0035\pm0.0015\ \mu^{-1}$.[1]

E. Short-range correlations

It is necessary to make a more accurate treatment of the pair wave function at small distances since the operator $\bar{Y}_1(\mu^*r)$ is sensitive to the wave function at $r<1$ fm. Probably the best theoretical ansatz one could do would be to replace the oscillator pair wave function ϕ_{nl} by a wave function defined in the integral equation,

$$\psi_{nl}=\Big(1-(1-P)\frac{1}{E-H_0}V\Big)^{-1}\phi_{nl} . \tag{3.8}$$

Here V is the two-nucleon interaction, H_0 is the oscillator Hamiltonian, E is a starting energy, and P is an approximation to the Pauli operator which projects off occupied states. The wave function defined in (3.8) is not normalized; it must be explicitly normalized when used to calculate operator matrix elements. The theory becomes very simple if $(1-P)$ only projects off the initial state ϕ_{nl} and E is chosen as the eigenenergy of the equation

$$(H_0+V)\psi^{nl}=E^{nl}\psi^{nl} . \tag{3.9}$$

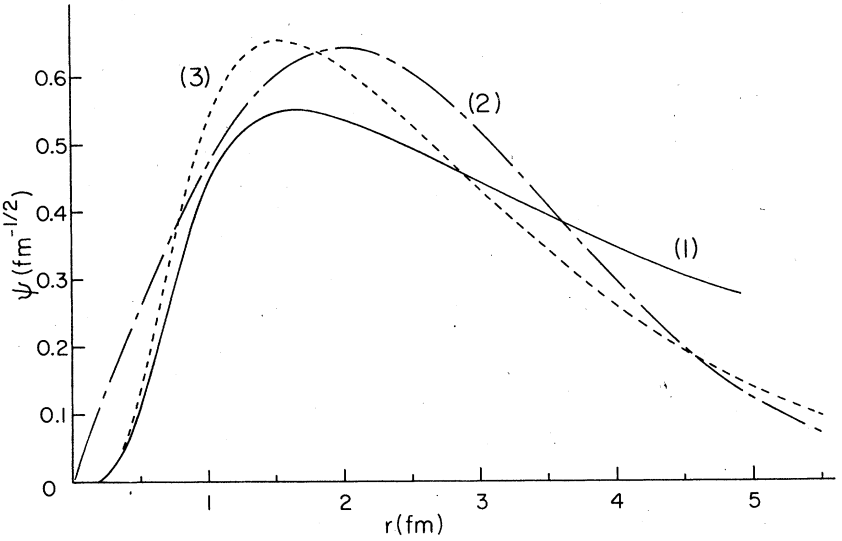

FIG. 2. (1) Deuteron S-state wave function for the Reid soft core potential, (2) Os oscillator function adjusted to the same rms radius as (1), (3) correlated Os eigenstate for Eq. (3.9).

Then the eigenfunction is equal to the Brueckner function, when the latter is normalized. We shall apply (3.9) to ^{4}He. In this nucleus the Pauli principle is indeed weak, and (3.9) should be an excellent approximation to the wave function. Again, we choose the oscillator Hamiltonian to reproduce the rms radius. Taking into account the different rms radii of $^1S_0$ and $^3S_1$-$^3D_1$ pairs, the oscillator frequency required is $\omega = 12.7$ MeV. The resulting $^3S_1$-$^3D_1$ wave functions are plotted in Figs. 2 and 3. For comparison, we also show the oscillator function having the same mean square radius, and the Reid soft core deuteron wave function.[13] We see that the pair in the α particle has an amplitude larger than that of the deuteron at small distances by about 30%.

Correlations will also be important in the final state. The final channel corresponding to the ^3S pair is the $^3P_1$ channel. The interaction is quite repulsive in this channel; the n-n phase shift at the energy of interest, 140 MeV in the c.m., is $-26°$. We calculate the wave function in the $^3P_1$ channel with Eq. (3.9), using Reid's soft core potential and a oscillator Hamiltonian with $\omega = 11$ MeV. The $(nl) = (51)$ eigenfunction is plotted in Fig. 4, together with the corresponding oscillator wave function. It may be seen that the amplitude is substantially reduced in the region $\lesssim 1$ fm, which is the region most sensitive to the absorption operator. The displacement of the first node of the eigenfunction corresponds well with the asymptotic phase shift. The radial integrals deduced from the eigenfunctions are shown in Table VI.

Comparing the correlated ^{4}He pair results with the deuteron result, we see that the ^{4}He matrix element is about 25% larger than the deuteron matrix element, as is to be expected from the larger amplitude of the ^{4}He pair wave in Fig. 2. Also,

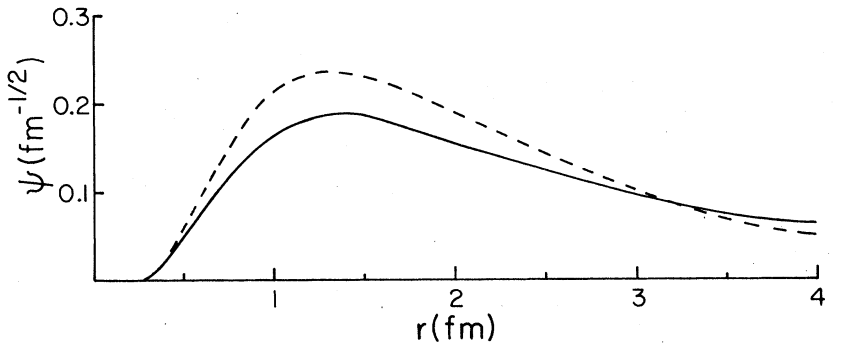

FIG. 3. Deuteron D wave function for the Reid soft core potential (solid line) and eigenstate corresponding to the $^3D_1$ state obtained from Eq. (3.9).

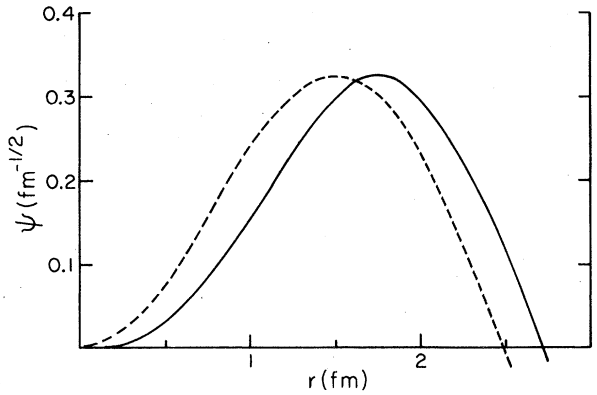

FIG. 4. (a) $5p$ eigenstate to Eq. (3.9), (b) $5p$ harmonic oscillator radial wave function (dashed line).

the relative contribution of S and D waves is nearly the same in the two cases. Thus, when correlations are included in the theory, the prediction will be that the ^{4}He rate be roughly 50% higher than what would be expected from the number of deuteron pairs in the nucleus.

Turning to the comparison of the correlated matrix elements with the oscillator matrix elements, we see that the short-range correlations reduce the matrix elements substantially. The effect of correlations in the $^3P_1$ is to reduce the matrix element by 20%. However, the coherence in the tensor correlations restores the 20% lost in the $^3P_1$ correlation. The net change on the matrix element is a 10% decrease. The result for the absorptive πd scattering length is $\mathrm{Im}a_d = 3.5 \times 10^{-3}$ μ^{-1} when the oscillator final state is replaced by a correlated scattering $^3P_1$ wave function, a result which is in perfect agreement with the empirical value.

As we saw in Sec. III D, the oscillator matrix elements predict somewhat too small an absorption rate, so the decrease caused by correlations makes the disagreement for heavy nuclei worse.

TABLE VI. The effect of short-range central and tensor correlations on $I_{00,51}$. The initial correlated wave function satisfies Eq(3.9) with $\omega_i = 0$ for the deuteron and $\omega_i = 12.7$ MeV for ^{4}He. For the ^{4}He pair the result is compared to an oscillator matrix element with $\omega_i = 20.7$ MeV. The final state wave function has $\omega_f = 11$ MeV in all cases.

Initial state	Final state	Deuteron	^{4}He
Oscillator S	Oscillator P		0.353
Correlated $^3S_1$	Oscillator P		0.333
Correlated $^3S_1$	Correlated $^3P_1$	0.204	0.256
Correlated $^3D_1$	Correlated $^3P_1$	0.074	0.093
$(^3S_1)+(1/\sqrt{2})$ $(^3D_1)$	Correlated $^3P_1$	0.264	0.322

Since the deuteron agrees with experiment, either our ideas on the pair correlations in complex nuclei are drastically wrong, or the spin-isospin pairs not present in the deuteron, such as $(ST) = (01)$, play a stronger role than predicted in the model. It is also plausible that some other dynamical absorption mechanism not considered so far could explain the remaining 20–30% discrepancy.

IV. IMAGINARY OPTICAL POTENTIAL

The imaginary part of the optical potential in infinite nuclear matter is of interest if the absorption reaction is to be represented by a local potential. We shall compute this potential in the Fermi gas model, and compare the result with the phenomenological value deduced by Krell and Ericson.[10]

The pair wave functions will now be plane waves. We may specify the spatial symmetry explicitly and write

$$\psi^{\pm}_{mn}(r_1, r_2) = \frac{1}{\Omega}\,\phi^{\pm}_{k_{mn}}(r) e^{i\vec{K}_{mn}\cdot\vec{R}}$$

$$= \frac{1}{\Omega}\frac{1}{\sqrt{2}}\,(e^{i\vec{k}_{mn}\cdot\vec{r}} \pm e^{-i\vec{k}_{mn}\cdot\vec{r}}) e^{i\vec{K}_{mn}\cdot\vec{R}}. \quad (4.1)$$

Here the radial variables are defined as $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$, $\vec{r} = \vec{r}_1 - \vec{r}_2$, and the momentum variables as $\vec{K}_{mn} = \vec{K}_m + \vec{k}_n$ and $\vec{k}_{mn} = \frac{1}{2}(\vec{k}_m - \vec{k}_n)$. The components of the ground state wave function are labeled by the momentum indices m, n, and the $(\pm)$ signs in (4.1) stand for symmetric and antisymmetric pair wave functions, respectively. The volume of the system, Ω, is assumed to be large and will drop out of the final formula for the absorption rate of a single pion.

The evaluation of the isospin matrix elements and spin matrix elements proceeds exactly as in Sec. III C and leads to an expression for the total matrix element $|T_{if}|^2$ in terms of the vector matrix elements $\vec{V}^{\alpha}_{if}$ analogous to Eq. (2.15). The matrix element of $\vec{V}^{\alpha}$, has the form

$$\vec{V}^{\alpha}_{if} = K^{\alpha} \int d^3r_1 d^3r_2 \psi^*_{mn} \hat{r}_{12} \overline{Y}_1(\mu^* r) \psi_{kl}\,. \quad (4.2)$$

As in Eq. (2.18), we shall assume that the final states form a complete set, so that the c.m. integral may be performed by specifying the c.m. wave function to be the same in the initial and the final state. Equation (4.2) then becomes

$$\vec{V}^{\alpha}_{if} = \Omega^{-1} K^{\alpha} \int d^3r \phi^*_{k_i}(\vec{r}) \hat{r} \overline{Y}_1(\mu^* r) \phi_{k_f}(\vec{r})\,. \quad (4.3)$$

After angular momentum decomposition of the relative wave functions,

$$\phi_k^{\pm}=\sqrt{2}\,4\pi \sum_{\substack{\text{even}\\ \text{odd}}\ L} i^L Y^{L*}(\hat{k})\cdot Y^L(\hat{r}) j_L(kr)\,, \tag{4.4}$$

it becomes possible to express V_{if} in terms of the radial integrals of the form

$$I^{LL'}_{k_i k_f}=\int r^2 dr j_L(k_i r) j_{L'}(k_f r)\bar{Y}_1(\mu^* r)\,. \tag{4.5}$$

The result for the μth spherical component of the vector $\vec{V}$ is

$$(\vec{V}^{\alpha}_{if})_\mu = 2(4\pi)^2\Omega^{-1}K^{\alpha}\sum_{\substack{LL'\\ MM'}} Y^L_M(\hat{k}_i)Y^{L'*}_{M'}(\hat{k}_f)\begin{pmatrix} L & L' & 1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} L & L' & 1\\ -M & M & \mu\end{pmatrix} i^{L'+L}(-1)^{M+L}(2L'+1)^{1/2}(2L+1)^{1/2} I^{LL'}_{k_i k_f}. \tag{4.6}$$

We now square the matrix element and sum over initial and final states. The sum over initial states may be replaced by an integral over the relative momentum of the initial pair:

$$\sum_i = \tfrac{1}{32}\Omega^2\rho^2\int_0^1 d\left(\frac{k}{k_f}\right)P\left(\frac{k}{k_f}\right)\int\frac{d\hat{k}}{4\pi}\,. \tag{4.7}$$

Here ρ is the density of the Fermi gas, so the coefficient of the integral is the total number of pairs. Also, k_F is the Fermi momentum, and $P(x)$ is a normalized weight function, given by

$$P(x)=24(1-\tfrac{3}{2}x+\tfrac{1}{2}x^3)x^2\,. \tag{4.8}$$

The sum over final states will be evaluated as

$$\sum_f \delta(E_f-E_i-\omega)=\tfrac{1}{2}\Omega\int\frac{d^3k_f}{(2\pi)^3}\,\delta\left(\frac{k_f^2}{m}-\frac{k_i^2}{m}-\mu\right) = \Omega\frac{mk_f}{4(2\pi)^3}\int d\hat{k}_f\,. \tag{4.9}$$

Here then $k_f=(k_i^2+m\mu)^{1/2}$ because of the δ function. The factor $\frac{1}{2}$ in the first line is necessary as the final states were defined with $\phi^{\pm}_{\hat{k}_f}=\pm\phi^{\pm}_{-\hat{k}_f}$.

The angular integrations for the square of the matrix element (4.6) yield,

$$\int\frac{d\hat{k}_i}{4\pi}\int d\hat{k}_f\,\vec{V}^{\alpha}_{if}\vec{V}^{\beta *}_{if} = \frac{K^{\alpha}K^{\beta}}{\Omega^2}2(4\pi)^3\sum_{LL'}(2L+1)(2L'+1) \times\begin{pmatrix} L & L' & 1\\ 0 & 0 & 0\end{pmatrix}^2 (I^{LL'}_{k_i k_f})^2\,. \tag{4.10}$$

The transition rate will then only require squares of radial integrals $I^{LL'}$ in the form

$$J^{LL'}=(2L+1)(2L'+1)\begin{pmatrix} L & L' & 1\\ 0 & 0 & 0\end{pmatrix}^2 \times\int_0^1 dx P(x)x'(I^{LL'}_{k_F x', k_F x'})^2\,, \tag{4.11}$$

with $x'=(x^2+\mu m/k_F{}^2)^{1/2}$. Carrying out the complete sum over all initial and final states finally leads to the expression

$$\sum_{if}\delta(E_f-E_i-\omega)\vec{V}^{\alpha *}_{if}\cdot\vec{V}^{\beta}_{if}=\tfrac{1}{2}mk_F\Omega\rho^2\sum_{LL'}J^{LL'}. \tag{4.12}$$

We insert this expression in (2.15), and use (2.4) and (2.25) to obtain a final equation for the scattering length parameter B_0:

$$B_0=\tfrac{9}{8}mk_F\Big([\lambda_1{}^2+(\lambda_1+\tfrac{3}{2}\lambda_2)^2]\sum_{L\,\text{even}}^{L'} + [\lambda_1{}^2+(\lambda_1-\tfrac{3}{2}\lambda_2)^2]\sum_{L\,\text{odd}}^{L'}\Big)J^{LL'}. \tag{4.13}$$

We evaluate this with $k_F=1.34\ \text{fm}^{-1}=1.9\ \mu$. The reduced integrals $J^{LL'}$ which contribute 1% or more to the absorption rate are tabulated in Table VII. Note that the S wave is again the most important initial pair configuration. As in the model of the light nuclei, the P wave is also significant; other pairs are negligible. Inserting the values of $J^{LL'}$ in Eq. (4.13), we find for the imaginary scattering length the value $\text{Im}B_0=0.031\ \mu^{-4}$, which should be compared with the phenomenological value derived from the pionic atom level widths $0.042\ \mu^{-4}$.[1] This 25% discrepancy is similar to that for the absorption lengths in light nuclei. As in the previous situation, we expect that consideration of the tensor correlations and short-range repulsion would respectively increase and decrease the result, leading to a small change in the end.

TABLE VII. Radial integrals for the Fermi gas model result (Eq. 4.11). Only integrals larger than $0.01\,\mu^{-6}$ are tabulated.

$J^{01}=0.270\ \mu^{-6}$
$J^{21}=0.003\ \mu^{-6}$
$J^{10}=0.022\ \mu^{-6}$
$J^{12}=0.064\ \mu^{-6}$
$J^{43}=0.003\ \mu^{-6}$

V. EXTENSION TO VECTOR MESONS

The fact that the predicted absorptions rates are too small for nuclei more complex than the deuteron suggests that other mechanisms besides pion

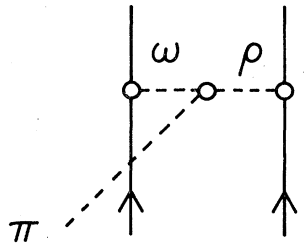

FIG. 5. Two-nucleon absorption mechanism involving a $\pi\rho\omega$ vertex.

rescattering may be of significance. One mechanism which will contribute to nuclei with $^1S_0$ pairs but not to the deuteron is the absorption into a ρ and ω meson vertex. This is illustrated in Fig. 5.

In order to construct the $\rho\omega$ absorption operator we use the following interaction Lagrangians for the $\omega N\bar{N}$, $\rho N\bar{N}$, and $\pi\rho\omega$ couplings:

$$L_{\omega N\bar{N}} = ig_\omega \bar{\psi}\left(\lambda_\mu + i\frac{\kappa^s}{2m}\sigma_{\mu\nu}\partial_\nu\right)\omega_\mu\psi\,,$$

$$L_{\rho N\bar{N}} = ig_\rho \bar{\psi}\left(\gamma_\mu + i\frac{\kappa^v}{2m}\sigma_{\mu\nu}\partial_\nu\right)\vec{\rho}_\mu\cdot\vec{\Omega}\psi\,, \tag{5.1}$$

$$L_{\pi\rho\omega} = i\frac{g_{\rho\omega\pi}}{m_\omega}\epsilon_{\mu\nu\gamma\delta}\partial_\mu\vec{\rho}_\nu\omega_\lambda\partial_\delta\vec{\phi}\,.$$

At threshold, the derivative of the pion field has only a timelike component. The antisymmetric tensor $\epsilon_{\mu\nu\lambda\delta}$ then gives rise to a triple cross product in the spacelike parts of the other vectors. Thus only the magnetic type coupling of the ρ and ω to the nucleons are present.

The graph of Fig. 5 [curve (1)] may thus be evaluated to give the result for the absorption operator at threshold,

$$T_{\rho\omega} = \frac{g_\rho g_\omega g_{\omega\rho\pi}}{4m^2}\frac{\tau_+^2}{\sqrt{\mu}}\frac{\mu}{m_\omega}(1+\kappa^v)(1+\kappa^s)\vec{k}^2 \times\frac{(\sigma^1\times\sigma^2)\cdot\vec{k}}{(k^2+m_\omega{}^2)(k^2+m_\rho{}^2)}\,. \tag{5.2}$$

Here $\vec{k}$ is the momentum of the exchanged ρ, with direction from nucleon 1 to nucleon 2.

The graph with particles 1 and 2 exchanged combined with (5.2) gives an operator with the isospin structure $(\tau^1+\tau^2)_+$. Thus this operator will not contribute to pion absorption in the deuteron.

After Fourier transforming the expression for the combined operator we find the result [in the notation of Eq. (2.10)]

$$t^1_{\rho\omega} = -i\frac{g_\rho g_\omega g_{\omega\rho\pi}}{\sqrt{\mu}\,16\pi}\left(\frac{\mu}{m_\omega}\right)\left(\frac{m_\omega{}^2}{m^2}\right) \times(1+\kappa^v)(1+\kappa^s)\frac{m_\omega{}^2}{m_\omega{}^2-m_\rho{}^2} \times\left(\bar{Y}_1(m_\omega r)-\frac{m_\rho{}^4}{m_\omega{}^4}\bar{Y}_1(m_\rho r)\right)(\sigma^1\times\sigma^2)\cdot\hat{r}\,. \tag{5.3}$$

Actually the argument in the Yukawa functions $\bar{Y}_1$ should involve $m_\rho^* = (m_\rho{}^2 - \frac{1}{4}\mu^2)^{1/2}$ and $m_\rho^* = (m_\rho{}^2 - \frac{1}{4}\mu^2)^{1/2}$ in analogy with the expressions for pion rescattering, but the smallness of the difference between the effective and bare vector meson masses makes the distinction insignificant.

The amplitude $t^1_{\rho\omega}$ will combine coherently with the pion rescattering amplitude t^1 given in Eq. (2.11). Writing

$$t_\rho = -i(\sigma^1\times\sigma^2)\cdot\vec{V}_{\rho\omega}\,, \tag{5.4}$$

we find that the expression for the square of the transition matrix element (2.15) is modified to

$$|T_{if}|^2 = 16\sum_f\left(\frac{1}{9}\sum_i^{11}(|\vec{V}^1_{if}+\vec{V}_{\rho\omega}|^2+|\vec{V}^2_{if}+\vec{V}^3_{if}|^2) +\frac{1}{3}\sum_i^{01}|\vec{V}^1_{if}-\vec{V}_{\rho\omega}|^2 +\frac{1}{3}\sum_i^{10}|\vec{V}^2_{if}-\vec{V}^3_{if}|^2\right). \tag{5.5}$$

when the $\rho\omega$ absorption mechanism is included.

From (5.5) it is straightforward to obtain the expression for the absorption length in finite nuclei by following the same steps that were elaborated in Sec. III. We need radial integrals of the form

$$\bar{I}_{nl,\bar{n}\bar{l}} = \int_0^\infty dr r^2\psi_{\bar{n}\bar{l}}(r)\phi_{nl}(r)\frac{m_\omega{}^2}{m_\omega{}^2-m_\rho{}^2} \times\left(\bar{Y}_1(m_\omega r)-\frac{m_\rho{}^4}{m_\omega{}^4}\bar{Y}_1(m_\rho r)\right), \tag{5.6}$$

in addition to those defined in Eq. (2.20) for pion rescattering. The expression for the absorption rate in Eq. (2.23) is modified to

$$\Gamma = \frac{32\pi}{3}f^2\frac{\mu^{*2}}{\mu^5}\sum_f\delta(E_f-E_i-\mu)\sum_{nl}[B^{10}_{nl}(\lambda_1+\tfrac{3}{2}\lambda_2)^2+\tfrac{1}{3}B^{11}_{nl}(\lambda_1-\tfrac{3}{2}\lambda_2)^2]\begin{pmatrix} l & \bar{l} & 1\\ 0 & 0 & 0\end{pmatrix}^2 I_{nl\bar{n}\bar{l}}{}^2 + \sum_n[B^{01}_{nl}(\lambda_1 I_{nl\bar{n}\bar{l}}-\lambda_{\rho\omega}\bar{I}_{nl\bar{n}\bar{l}})^2+\tfrac{1}{3}B^{11}_{nl}(\lambda_1 I_{nl\bar{n}\bar{l}}+\lambda_{\rho\omega}\bar{I}_{nl\bar{n}\bar{l}})^2]\begin{pmatrix} l & \bar{l} & 1\\ 0 & 0 & 0\end{pmatrix}^2, \tag{5.7}$$

when the $\rho\omega$ effect is included. The parameter $\lambda_{\rho\omega}$ in (5.7) is readily obtained from the expression (5.3) as

$$\lambda_{\rho\omega}=\frac{g_\rho g_\omega g_{\omega\rho\pi}}{12\pi f}\left(\frac{\mu}{m_\omega}\right)\left(\frac{m_\omega}{m}\right)^2(1+\kappa^v)(1+\kappa^s). \quad (5.8)$$

The magnitude of the $\rho\omega$ effect depends crucially on the values of the coupling constants in the expression for $\lambda_{\rho\omega}$, none of which is known with any degree of accuracy. For g_ρ and κ^v we use the values obtained by Höhler and Pietarinen:[15] $g_\rho^2/4\pi = 0.55$, $\kappa^v = 6.6$. These values are considerably larger than the earlier standard values $g_\rho^2/4\pi = 0.52$ and $\kappa^v = 3.7$ (obtained with the assumption that the hadronic vector current is proportional to the electromagnetic current). The $\omega N\bar{N}$ couplings are known with far less certainty; by analogy with the electromagnetic isoscalar current κ^s is usually taken to be -0.12. We shall assume $\kappa^s = 0$ for simplicity. According to SU_6, $g_\omega = 3g_\rho$ but the fits with realistic meson exchange models for the nucleon-nucleon interaction suggest much larger values: $g_\omega^2/4\pi \gtrsim 10$.[16] For definiteness we use the value $g_\omega^2/4\pi = 10$ which is favored in one of the most sophisticated meson-exchange models for the nucleon-nucleon interaction.[17]

The magnitude for the $\pi\rho\omega$ coupling constant $g_{\pi\rho\omega}$ is also not very well established. From the ρ-dominance model for the $\pi\pi\pi$ decay of the ω of Gell-Mann, Sharp, and Wagner,[18] and with their value for the $\rho\pi\pi$ coupling constant we can determine $g_{\pi\rho\omega}^2/4\pi \simeq 8$. On the other hand Miyata[19] using a dual model for an analysis of $\pi N \to \omega N$ and $\pi N \to \omega\Delta$ amplitudes finds $g_{\pi\omega}^2/4\pi = 14.4$. Finally Sakita and Wali[20] using a U(6, 6) symmetry scheme find $g_{\omega\rho\pi} = -2g_{\rho\pi\pi}$ (usual sign convention). Since $g_{\rho\pi\pi} = 2g_{\rho NN}$ in the universal vector current model, this relation gives $g_{\omega\rho\pi} = -10.5$ or $g_{\omega\rho\pi}^2/4\pi = 8.8$. An advantage of the relation for $g_{\pi\rho\omega}$ of Ref. 17 is that it determines the sign of the coupling constant. For definiteness we shall use the value $g_{\pi\rho\omega} = -10.5$ as obtained with this relation. With these coupling constant values we obtain $\lambda_{\rho\omega} = -7.7$, a number which must be regarded to be associated with an uncertainty of the order of 50%.

In Table VIII we list the values of the radial integrals $\bar{I}$ defined in Eq. (5.6) as obtained with oscillator wave functions with the same parameters as in Table II. The integrals $\bar{I}$ are much smaller in magnitude than those for pion rescattering because of the short range of the absorption operator. With the value $\lambda_{\rho\omega} = -7.7$ we find that the absorption length for the α particle is increased by 2.5% from the value obtained with pion rescattering alone. With the larger value for $g_{\pi\rho\omega}$ of Ref. 16 the increase would be 4%. Depending on the value for the $\omega N\bar{N}$ coupling constant the enhancement could possibly be as large as 6%, but probably not larger. It must be pointed out that in contrast to the pion rescattering mechanism the $\rho\omega$-absorption mechanism will be very sensitive to short-range correlations in the wave functions and thus the estimates above should be regarded as upper limits.

VI. DISCUSSION AND CONCLUSIONS

We have shown that 70–90% of the pion absorption rates in nuclei deduced from pionic atom level widths may be explained by the two-nucleon absorption mechanism involving pion rescattering. After having taken into account the tensor correlations in the ground state wave functions and the final state interactions we draw the conclusion that it is very unlikely that the remaining discrepancy would be caused by deficiencies in the wave function models.

The fact that the pion rescattering model is able to explain the empirical value for the πd scattering length perfectly suggests that the 10–20% discrepancy in nuclei other than the deuteron is due to some absorption mechanism that does not act in the deuteron. As an example of such a mechanism we consider pion absorption into a ρ and an ω meson, but find that this mechanism accounts for only part of the discrepancy. It is possible that absorption mechanisms involving more than two nucleons of the type illustrated in Fig. 6 may account for the remaining discrepancy.

In the first realistic calculation of pion absorption in nuclei more complicated than the deuteron Koltun and Reitan[8] also considered a specific one-nucleon absorption mechanism. While that mechanism was found to be of little importance in the

TABLE VIII. Values for the $\rho\omega$-absorption radial integrals $\bar{I}$ defined in Eq. (5.6) for oscillator wave functions with $\omega_i = 15.7$ MeV and $\omega_f = 11$ MeV.

$\bar{I}_{nl,\bar{n}\bar{l}}$	nl =	00	10	02	01	01
	$\bar{l}$ =	1	1	1	0	2
$\bar{n} = 5$		0.302-3	0.414-3	−0.283-4	0.217-3	−0.554-4
$\bar{n} = 6$		0.360-3	0.482-3	−0.261-4	0.242-3	−0.584-4
$\bar{n} = 7$		0.420-3	0.551-3	−0.235-4	0.263-3	−0.602-4

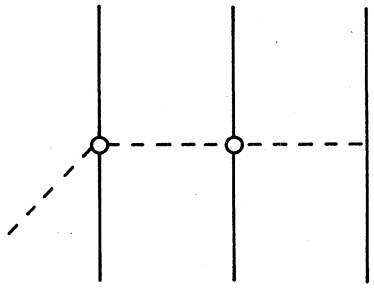

FIG. 6. Three-nucleon absorption mechanism involving double rescattering.

deuteron because of cancellations between matrix elements involving the S- and D-state components of the deuteron wave function it was found to contribute large amplitudes in the case of ^{4}He, with a consequent 50% overestimate of the absorption rate.[9] Since the question of the proper form of the threshold one-body absorption operator has not been settled in the absence of a fundamental theory of hadron dynamics and since there is no compelling empirical avidence in favor of it we have preferred not to consider one body absorption.

One convenient alternative way to approach one-nucleon absorption would be to consider rescattering diagrams in which mesons other than the pion are exchanged and where the relativistic nucleon propagator is used in the intermediate state at the rescattering vertex. This approach which would take into account the binding effects to lowest order would probably lead to small matrix elements because of the short range of the operators involved.

We finally turn to the question of the ultimate validity of the zero-range $\pi N\bar{N}$ interaction (2.2) which we used to construct the pion rescattering operator in this paper. The main argument in favor of the interaction model is the fact that it is the simplest model that can represent the S-wave pion-nucleon scattering phase shifts below 50 MeV adequately. Despite this argument it is clearly of interest to investigate the consequences of alternative and more fundamental models for the rescattering operator.

The usual dynamical picture of the isovector term in the Hamiltonian (2.2) is the $\pi\rho$ absorption mechanism illustrated in Fig. 7(a). If one chooses the relatively large value $g_\rho^2/4\pi = 0.77$ for the ρ-nucleon coupling constant the ρ-exchange model agrees with the S-wave scattering length combination $a_1 - a_3$. The pion rescattering operator associated with the Feynman diagram in Fig. 7(a) takes the form

$$T_{\pi\rho} = -i\frac{2fg_\rho^2\sqrt{2}}{(2\omega_{\vec{q}})^{1/2}}\frac{\sigma^2\cdot\vec{k}(\omega_{\vec{q}}+\omega_{\vec{k}})}{(\mu^2+\vec{k}^2-\omega_{\vec{k}}^2)(m_\rho^2+\vec{k}^2-\omega_\rho^2)} \times(\tau^1\times\tau^2)_+, \tag{6.1}$$

which should be compared to the zero-range model result (2.5). In the limit when $m_\rho \to \infty$ the expressions agree provided $g_\rho^2/4\pi = \frac{1}{2}\lambda_2(m_\rho^2/\mu^2)$. With the assumption that $\omega_{\vec{k}} = \omega_\rho$ (the energy of the ρ meson) $= \frac{1}{2}\mu$ it becomes simple to Fourier transform the operator. The net effect will be to modify the radial integrals in Eq. (3.5) by the replacement

$$Y_1(\mu^* r) \to Y_1(\mu^* r) - \frac{m_\rho^2}{\mu^{*2}} Y_1(m_\rho r). \tag{6.2}$$

The finite-range modification (6.2) would appear to be appreciable unless the wave function correlations are taken into account. For example the radial integrals with oscillator wave functions for ^{12}C are reduced by on the average 15% with the replacement (6.2). Using oscillator wave functions to evaluate the matrix elements of short-range operators will, however, always lead to gross overestimates of short-range effects. This is best illustrated by considering the finite-range effect in the deuteron. Using the Reid soft core deuteron wave functions and for the $^3P_0$ final scattering state the solution obtained with the Reid soft core potential leads to a reduction of only 2% of the two deuteron integrals I in Eq. (3.6). This suggests that the finite-range effect should be rather small—

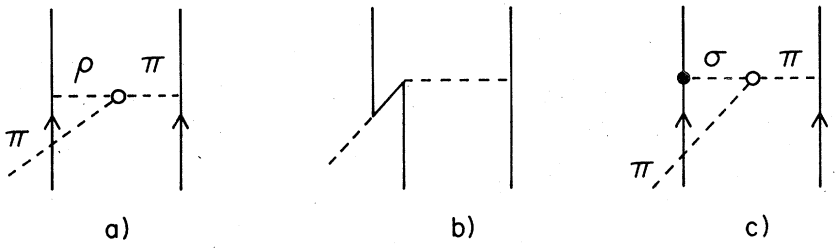

FIG. 7. (a) Pion absorption into a π and a ρ meson, (b) pion absorption involving a nucleon-antinucleon pair, (c) pion absorption into a pion and a correlated S-wave $\pi\,\pi$ state (σ meson).

probably affecting the absorption rate by less than 10%.

It is much more difficult to construct a realistic dynamical model for the isoscalar term in the Hamiltonian (2.2). This term arises from a combination of the diagrams in Figs. 7(b) and 7(c), one involving a correlated two-pion intermediate state often described by a fictitious σ meson, and the other involving a $N\bar{N}$ pair in the intermediate state. These two mechanisms are individually large but cancel almost completely leaving the very small value for λ_1. While it has long been common to approximate the correlated S-wave $\pi\pi$ state by a single σ meson in one-boson-exchange models of the nuclear force, such a representation is, in fact, but a poor one. While the pair diagram in Fig. 7(c) is well approximated by a zero-range interaction a diagram involving a σ meson would demand a finite-range correction. With a single σ meson the finite-range correction makes it impossible to maintain the near cancellation between the two mechanisms off shell, and the effective value for λ_1 in the rescattering amplitude would be much enhanced over that appropriate for the physical πN scattering amplitude. If the σ-meson mass is of the order of the ρ-meson mass we estimate that the value for λ_1 should be increased by a factor of the order 10. Such an enhancement would make it impossible to obtain a realistic value for the πd absorption rate, which should be the crucial test for any model for nuclear pion absorption.

In a recent manuscript Hachenberg and Pirner[21] employ a relativistic phenomenological Lagrangian which also contains nucleon and Δ_{33} intermediate states to describe the rescattering vertex. Their values for the absorptive part of the πd scattering length and for $\mathrm{Im}B_0$ are remarkably close to ours, despite the fact that their off-shell πN scattering amplitude has an enormously enhanced effective value for λ_1 and a correspondingly reduced value for λ_2. Thus it seems that the total absorption rates are rather insensitive to the off-shell behavior of the πN scattering amplitude.

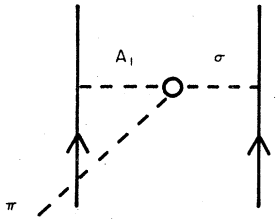

FIG. 8. Pion absorption involving a scalar (σ) and axial vector meson (A_1).

One might consider introducing hadronic form factors at the pion nucleon vertices in the rescattering operator (2.8). Recent results for the range of the pionic form factor of the nucleon indicate that it is smaller than $1/m \simeq 0.15$ fm.[22] Because of the long range of the pion exchange operator such form factors would therefore reduce the matrix elements in the expression for the absorption rate only slightly. We emphasize that our results do not allow long-range form factors of the type used, for example, in Ref. 3.

In addition to the pion rescattering and $\rho\omega$-absorption mechanisms that we have considered in detail one could also think of many other two-body absorption processes involving heavy meson exchange. As such processes would be associated with short-range operators their effects must be small because of the wave function correlations. The possibly most important such process of that type is the σA_1-exchange mechanism illustrated in Fig. 8. Here again the σ meson represents the correlated S-wave $\pi\pi$ state. As this particular process would act as well in the deuteron as in heavier nuclei we assume that its magnitude must be small since we have little room for an appreciable effect in the calculated value for the πd absorption rate. We therefore find that the most likely explanation for the fact that the calculated nuclear absorption rates are 10–20% too small in contrast to the deuteron result is some many-body absorption process of the type illustrated in Fig. 6.

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