Measuring short-range correlations (between nucleons)

1976 J. Phys. G: Nucl. Phys. 2 L121

(http://iopscience.iop.org/0305-4616/2/8/004)

J. Phys. G: Nucl. Phys., Vol. 2, No. 8, 1976. Printed in Great Britain. © 1976

LETTER TO THE EDITOR

Measuring short-range correlations†

G Bertsch and J Borysowicz
Physics Department and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA

Received 4 June 1976

Abstract. We re-examine the problem of determining short-range correlations by sum rules for cross sections at constant momentum transfer. Using the model-independent analysis technique, we find that knowledge of the summed cross sections for momentum transfers 3 fm^{-1} to 4·5 fm^{-1} is crucial for observation of short-range repulsion. Unfortunately, meson production will obscure the interpretation of the data in just this region.

The character of the short-range interaction between nucleons is basic to nuclear theory. The most frequently used model, the potential of Reid (1968), has strong repulsion at short distance, as do many other models. At one point measurement of the resulting correlation was contemplated (McVoy and Van Hove 1962), but rejected as experimentally not possible. We here re-examine this old question, using model-independent analysis of theoretically generated 'data'. The conclusion of McVoy and Van Hove is affirmed by our work, but we are able to provide a more quantitative specification of the requirements of the measurement. Specifically, the experimental requirements are

(*a*) an electron beam of > 1·5 GeV energy,
(*b*) data out to momentum transfers of 4·5 fm^{-1} and
(*c*) precision of 1%, relative to the cross sections on hydrogen and deuterium.

However, meson production will cause uncertainties of the order of 1%, thus ruling out unambiguous interpretation of an experiment which measures only the final electron.

The basic theoretical construct we consider is the structure function

$$S(q) = \langle 0| \sum_{i<j} \exp [\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)] |0\rangle.$$

Its Fourier transform is the desired two-body correlation function

$$\rho(r_{12}) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 q \, \frac{S(q)}{\binom{N}{2}}.$$

On the experimental side, we will consider a simple system, the ^{4}He nucleus, and measurement of the difference in cross sections between helium and deuterium:

$$I(q) = \int_{E-\Delta E}^{E} \mathrm{d}E' \, (\sigma_{\mathrm{He}}(q, E') - 2\sigma_{\mathrm{d}}(q, E'))$$

† Work supported in part by NSF, USA.

where $\sigma(q, E')$ is the electron scattering cross section $d\sigma/d\Omega\, dE'$ at momentum transfer q and with final electron energy (laboratory) of E'. The initial electron energy is E and the cross section is integrated over an energy interval ΔE. Absolute measurements of electron scattering have difficulty attaining a precision of 1% because of the bremsstrahlung corrections. However, bremsstrahlung contributions roughly factorize, so that the error in the expression in brackets is not worsened by the uncertainty in the correction. Relative measurements at lower momentum transfers are reported to $\frac{1}{2}$% accuracy (Euteneuer *et al* 1976).

If we only consider Coulomb scattering, or more generally, the longitudinal form factors, the structure function of the two protons in the alpha particle is half the ratio of the difference in cross sections $I(q)$ to the proton cross section:

$$S^{pp}(q) = \frac{I(q)}{2\int_{E-\Delta E}^{E} \sigma_p(q, E')\, dE'}. \qquad (1)$$

$S(q)$ will have the limits

$$S(q) \to 1 \qquad \text{as } q \to 0$$
$$S(q) \to 0 \qquad \text{as } q \to \infty .$$

We first discuss how well $S(q)$ must be determined for a clear demonstration of short-range repulsion. We then argue that corrections to the simple formula (1) from transverse scattering can be estimated well enough for the purpose of demonstration of correlations. Finally, we examine the energy cut-off on the sum rule and find that the uncertainty which this introduces is as large as the effect being measured.

Model-independent analysis

A density distribution in coordinate space is generated from a structure function by assuming a finite basis set of functions and making a χ^2 fit of the structure function to the Fourier transform of functions in this finite basis. If the basis is sufficiently large, the envelope of the set of functions that fit equally well does not depend on the particular finite set, *providing* that the high-q behaviour of the structure function is specified. This is what we mean by model-independent analysis (Hetherington and Borysowicz 1974).

To check our ability to recover the correlation function from experiment we performed numerical 'experiments'. Starting with a hypothetical correlation function we produced an 'experimental' structure function by adding random (normally distributed) errors to the exact form factor for q less than certain q_{max}. We assumed that nothing is known for $q > q_{max}$. We then applied our analysis to the 'data' so obtained.

We found that we can predict the gross behaviour of the correlation function at short distances if the structure function is known with 1% accuracy for $0 \leqslant q \leqslant 2{\cdot}5\ \text{fm}^{-1}$ and with 3% accuracy for $2{\cdot}5\ \text{fm}^{-1} \leqslant q \leqslant 4{\cdot}5\ \text{fm}^{-1}$, i.e.

$$\begin{aligned} \sigma(S^{exp}(q)) &= 0{\cdot}01 \qquad 0 < q < 2{\cdot}5 \\ &= 0{\cdot}03 \qquad 2{\cdot}5 < q < 4{\cdot}5. \end{aligned} \qquad (2)$$

One example of a correlation function is that obtained with a Reid potential in the $^1S_0$ channel. In addition a harmonic potential is superimposed with strength adjusted to give 1·5 fm RMS radius for ^{4}He. The exact structure function corresponding to this

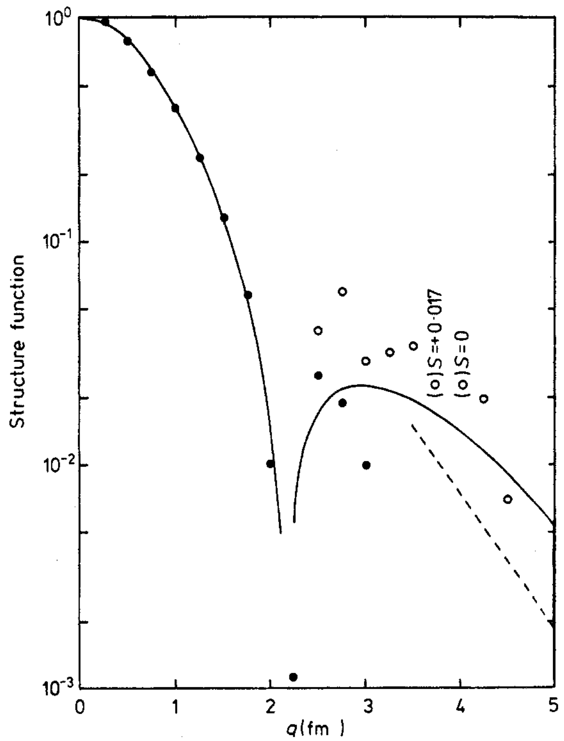

Figure 1. Structure function appropriate to the α particle derived from the Reid interaction and an oscillator potential. 'Experimental' data were obtained by adding random quantities to the values of the structure function at the points indicated. The variance of the errors introduced are 0·01 for the full points and 0·03 for the open points. The broken curve is the upper bound used in the analysis with the cut-off of 3 fm^{-1}.

correlation function is shown as the full curve in figure 1 and the 'experimental' one as the points in the figure. As in the previous work (Hetherington and Borysowicz 1974) we assume an upper bound for the structure function

$$|S(q)| \leqslant 0{\cdot}01 \times 10^{-0{\cdot}5(q-5{\cdot}0)}$$

in the region $5\ \mathrm{fm}^{-1} \leqslant q \leqslant 9\ \mathrm{fm}^{-1}$.

The resulting fit to the correlation function with the set of functions

$$f_n = r^n \mathrm{e}^{-1{\cdot}73r} \qquad n = 1,8$$

is shown in figure 2 as the broken curve. The curve does not match the exact structure functions, primarily because of the randomness introduced in the 'experimental' structure function. The error bars in the figure are the diagonal matrix elements of the inverse of the error matrix in coordinate space. This is obtained from the specified errors in the structure function (equation (2)).

When we assumed that the structure function is known only for $q \leqslant 3$ (1% accuracy) our predictions were usually wrong. This is illustrated in figure 3, for which we used the 'experimental' structure function for $q \leqslant 3$, and the plausible guess

$$|S(q)| \leqslant 0{\cdot}015 \times 10^{-0{\cdot}58(q-3{\cdot}5)}$$

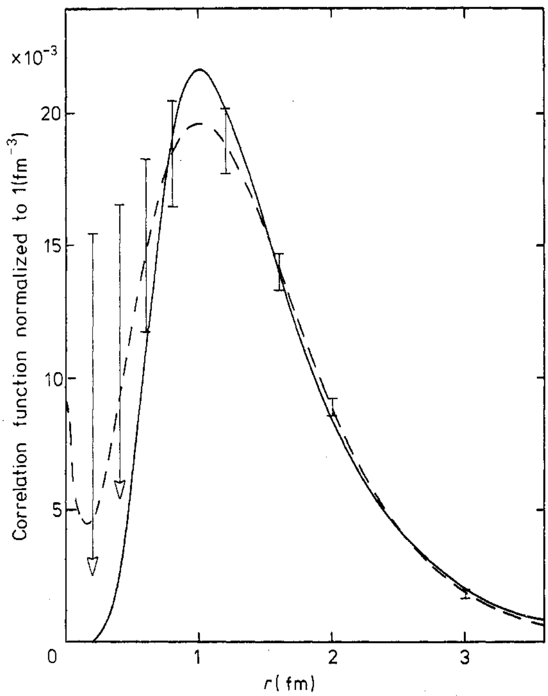

Figure 2. Fit (broken curve) to the exact correlation function (full curve) with the 'experimental' data given for $0 < q < 4{\cdot}5\ \mathrm{fm}^{-1}$ and errors described in the text.

for $q \geqslant 3$. In fact, the structure function has a secondary maximum in the region $2{\cdot}5 \leqslant q \leqslant 4$, and this guess contains misinformation. Therefore, knowledge of the structure function in the region $3 \leqslant q \leqslant 4{\cdot}5$ is necessary to determine the correlation function for $0{\cdot}5 < r < 1$. It was also sufficient to discriminate between different correlation functions for the several cases we tried.

Theoretical corrections

The corrections to the Coulomb model (equation (1)) are due to magnetic scattering, the finite energy cut-off on the sum and mesonic effects.

To discuss the magnetic scattering, we display the theoretical cross section for unpolarized targets. In the notation of McVoy and Van Hove (1962) this cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{0\to n}} = \frac{e^4}{4k_1^2 \sin^4 \frac{1}{2}\theta_{0n}}\left[\frac{\cos^2 \frac{1}{2}\theta_{0n}\, Q_{0n}^* Q_{0n} + \frac{1}{2}(\cos^2\frac{1}{2}\theta_{0n} + 2\sin^2\frac{1}{2}\theta_{0n}) J_{0n}^* J_{0n}}{(1 + 2E/M \sin^2 \frac{1}{2}\theta)}\right]$$

where

$$Q_{0n} = \left\langle 0 \left| \sum_j \exp(\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{r}_j)\left[F_1(1+\tau_{3j})\frac{1}{2}\left(1 - \frac{q^2}{8M^2}\right) - \frac{q^2}{4M^2} K F_2 \tau_{3j}\right]\right| n \right\rangle$$

and

$$\boldsymbol{J}_{0n} = \left\langle 0 \left| \sum_j \frac{\frac{1}{2}F_1(1+\tau_{3j})}{2M}(\boldsymbol{p}\exp(\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{r}_j) + \exp(\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{r}_j)\boldsymbol{p}) \right.\right.$$

$$\left.\left. + \frac{\frac{1}{2}F_1(1+\tau_{3j}) + KF_2\tau_{3j}}{2M}\mathrm{i}\boldsymbol{\sigma}_j \times \boldsymbol{q}\exp(\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{r}_j)\right| n \right\rangle.$$

Following McVoy and Van Hove (1962), we sum over intermediate states and evaluate the sum by closure.

Closure can only be invoked if the variation of angle θ_n with excitation energy is neglected, or specific corrections are made. With this approximation, the operators in the products QQ and JJ can be multiplied directly. One-body and two-body parts are separated out; the one-body parts do not depend on r, and in fact cancel in the cross section difference I.

We are left with two-body terms only, but with a substantial magnetic contamination to the Coulomb structure function.

We now must assume that the wavefunction of the alpha particle is symmetric in space and has zero spin and isospin and that the magnetic term has the same structure function as the electric term.

Then, except for the small term in J containing $\boldsymbol{p}$, the structure function factors out of the two-body matrix elements completely. Neglecting terms with $\boldsymbol{p}$, we find

$$\int \sigma_\alpha(q, E)\,\mathrm{d}E = \text{one-body terms} + S(q)\frac{e^2}{2k_1^2\sin^4\frac{1}{2}\theta}\left\{\left[F_1^2\left(1-\frac{q^2}{8M^2}\right)^2 + 2F_1\left(1-\frac{q^2}{8M^2}\right)\frac{q^2}{4M^2}KF_2\left(-\frac{q^4}{8M^2}K^2F_2^2\right)\right]\cos^2\tfrac{1}{2}\theta - 2(\sin^2\tfrac{1}{2}\theta+\tfrac{1}{2}\cos^2\tfrac{1}{2}\theta)q^2\left[\left(\frac{F_1+KF_2}{2M}\right)^2 + \left(\frac{KF_2}{2M}\right)^2\right]\right\}[(1+2E/M_\alpha\sin^2\tfrac{1}{2}\theta)]^{-1}$$

$$2\int \sigma_\mathrm{d}(q, E)\,\mathrm{d}E = \text{one-body terms} + S'(q)\frac{e^2}{2k_1^2\sin^4\frac{1}{2}\theta} \times\left[\left(F_1-\frac{q^2}{8M^2}(F_1+2KF_2)\right)\frac{-q^2}{8M^2}2KF_2\cos^2\tfrac{1}{2}\theta + \tfrac{2}{3}(\sin^2\tfrac{1}{2}\theta + \tfrac{1}{2}\cos^2\tfrac{1}{2}\theta)\left(-\frac{KF_2}{2M}\frac{F_1+KF_2}{2M}q^2\right)\right][(1+2E/M_\mathrm{d}\sin^2\tfrac{1}{2}\theta)]^{-1}.$$

We desire to minimize the importance of the second term in brackets in the formula for σ_α. This requires that $\sin^2\theta \lesssim \frac{1}{2}\cos^2\theta$, which can be satisfied for quasi-elastic scattering at $q = 4{\cdot}0\ \mathrm{fm}^{-1}$ if the incident electron beam has an energy of the order of 1·5 GeV. The form factors in these equations have the values shown in table 1 at the higher momentum transfers (Bumiller *et al* 1961).

Magnetic corrections will thus be of the order of 100% of the measured $S(q)$ between 3 and 4·5 fm^{-1}. These will be tolerable if the values and errors of the empirical form factors are trusted.

Next we consider meson production and the choice of the upper limit on the cross section integral. The first point we wish to make is that the cross section associated with copious meson production, namely the Δ peak lying about 250 MeV above the quasi-elastic peak, must be excluded from the sum rule. Whatever correlations give rise to nonvanishing $I(q)$ from quasi-elastic scattering also give

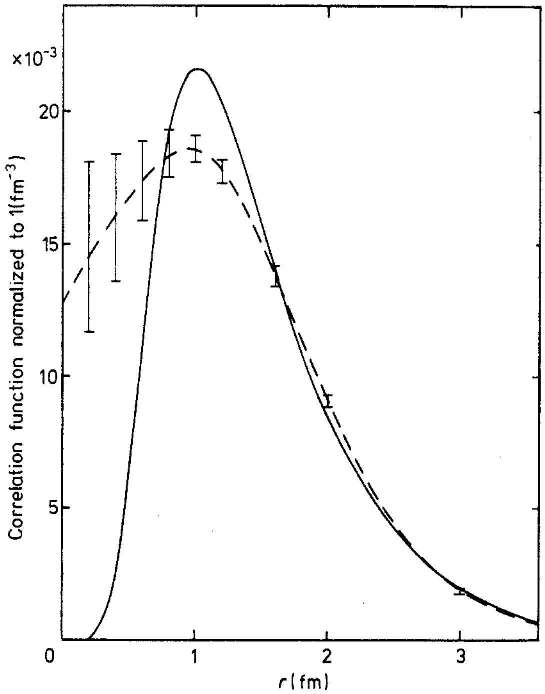

Figure 3. Fit (broken curve) to the exact correlation function (full curve) with the 'experimental' data given for $0 \leqslant q \leqslant 3$.

differing meson production cross sections. Since meson production is more probable at 4·5 fm^{-1} than elastic scattering, $I(q)$ would be dependent mostly on mesonic effects.

If the energy cut-off in the sum rule is made lower, say at 200 MeV above the quasi-elastic peak, significant error will be introduced from missing the tail of the quasi-elastic peak. The shape of the quasi-elastic peak can easily be calculated in an independent-particle model (McVoy and Van Hove 1962). With a Gaussian wavefunction that fits the RMS radius of the α particle, roughly 2% of the sum rule would be missed at $q = 4{\cdot}5\ \mathrm{fm}^{-1}$, with an energy cut-off of 200 MeV. Also, short-range correlations will certainly have an effect on the final state energy. The central

Table 1.

	$q(\mathrm{fm}^{-1})$	
	3·0	4·5
$F_1^2(q)$	0·25	0·13
$F_1^2\left(1-\dfrac{q^2}{2M^2}\right)^2$	0·21	0·10
$\dfrac{q^2}{4M^2}(KF_2)^2$	0·055	0·013
$\dfrac{q^2}{4M^2}(F_1+KF_2)^2$	0·15	0·08
$\dfrac{q^2}{4M^2}(2KF_1F_2)$	0·07	0·04

part probably does not affect the energy distribution of the final state very much, since these correlations are the same in initial and final state. The tensor correlations, because of their spin and isospin dependence, do change drastically in the final state. We can estimate the effect in the deuteron by calculating the kinetic energy in the D state. We find that the probability of the deutron having D-wave kinetic energy greater than 200 MeV is about 1%. Thus the individual cross sections in $I(q)$ would be uncertain by this amount. For the α particle we were unable to find estimates of the tensor correlations, but we might expect the high-momentum component to be somewhat larger.

In summary, we affirm the result of McVoy and Van Hove that an unambiguous measurement of the short-range correlations is not feasible by a study of the (e, e′) cross section, because of the nearness of the meson threshold. If meson production could be excluded from the measurement, the main difficulty in the interpretation of the data would be the required assumption that the correlation function was the same for all nucleons in the α particle.

References

Borysowicz J and Hetherington J H 1973 *Phys. Rev.* C **7** 2293
Bumiller F, Croissiaux M, Dally E and Hofstadter R 1961 *Phys. Rev.* **124** 1623
Euteneuer H, Friedrich J and Voegler N 1976 *Phys. Rev. Lett.* **36** 129
Hetherington J H and Borysowicz J 1974 *Nucl. Phys.* A **219** 221
McVoy K W and Van Hove L 1962 *Phys. Rev.* **125** 1034
Reid R 1968 *Ann. Phys., NY* **50** 411