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# Two-body short-range correlations and Coulomb matrix elements*

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Correlated Coulomb matrix elements are calculated by evaluating the appropriate $G$ matrices $G_{N+C}$ and $G_N$ which correspond to $V_N + V_C$ (nuclear + Coulomb) and $V_N$ potentials, respectively. Including the effect of the Pauli operator, we confirm the recent report by McCarthy and Walker that the effect of the two-body short-range correlations is small.

Very recently McCarthy and Walker (MW) have considered[1] the effect of two-body short-range correlations on the matrix elements of the Coulomb interaction within the nuclear $1p$ shell ($A \sim 6$). The correlated Coulomb matrix elements were calculated by solving the Bethe-Goldstone equation with Hamada-Johnston potential using the method of Barrett-Hewitt-McCarthy[2] (BHM). Including the Pauli projection operator, they found, contrary to the claim by the previous calculations of Anderson-Wilson-Goldhammer[3-5] (AWG), that the effect of short-range correlations is too small to account for the experimentally deduced Coulomb matrix elements. It is important to indicate here that the effect of the Pauli operator was neglected in the calculations of AWG.

In the calculations of MW and AWG the (correlated) two-particle Bethe-Goldstone wave functions with the nuclear potential $V_N$ are evaluated and then the Coulomb matrix elements are found by means of perturbation expansion. In order to avoid the consideration of the perturbation expansion, we evaluate in the present work the $G$ matrices $G_{N+C}$ and $G_N$ which correspond to $V_N + V_C$ (nuclear +Coulomb) and $V_N$ potentials, respectively. The difference $G_{N+C} - G_N$ is then taken to be the correlated matrix element of the Coulomb interaction $V_C$. The BHM method with the Eden-Emery[6] approximation for the Pauli operator is adopted for the evaluation of the $G$ matrix. In this work, we use the Reid-soft-core[7] potential which seems to give the largest correlation effect in the calculations of AWG.

In Table I we demonstrate our results for the Coulomb interaction $e^2/r$ and $\hbar\omega = 16.2$ MeV. The relative starting energy is taken to be $W_r = E_{nl} - E_D$, where $E_{nl}$ is the relative energy and $E_D$ is the constant indicated in Table I (i.e., $E_D$ is the energy denominator). We denote by $V_{nl}$ the Coulomb matrix element in the relative state $nl$ ($T=1$). The Coulomb integrals $L$, $K_{SD}$, and $K_{SP}$ which enter in the calculations of Coulomb energies in the $1p$ shell are defined in Refs. 1 and 3. For comparison, we give the values obtained using harmonic oscillator wave functions, correlated wave functions derived by solving the appropriate Schrödinger equations which include the Reid-soft-core potential and the values obtained using BHM method and including the Pauli projection operator. For the relative $1p$ state we quote the values $\bar{V}_{1p}$ obtained by a $(2J+1)$ weighted average of the $T=1$ $J=0, 1, 2$ matrix elements. The values obtained are equal within 2 keV to the appropriate matrix elements for $J=0$ and the difference between those of $J=2$ and $J=1$ is ~40 keV. We see from Table I that there is a relatively strong dependence on the starting energy. We note that one can consider the values $E_D = 50 \pm 20$ MeV in order to obtain an estimation for the theoretical errors due to the uncertainty in the starting energy.

Comparing our results with those of MW and AWG (Refs. 1 and 3, respectively) we find a complete agreement with the conclusion of McCarthy and Walker in Ref. 1. The correlated Coulomb matrix elements do not differ significantly from the uncorrelated ones. This is so because the enhancement of the relative wave function at small distances is compensated by the shorter-range

TABLE I. Matrix elements in keV of the Coulomb interaction $e^2/r$ for the nuclear $1p$ shell with $\hbar\omega = 16.2$ MeV are given. The Reid-soft-core potential and the BHM method are used to deduce the effect of the two-body short-range correlations. The starting energy is $W_r = E_{nl} - E_D$ where $E_{nl}$ is the relative energy (i.e., energy denominator $E_D$), $V_{nl}$ is the Coulomb matrix element in the relative $nl$ state, and $L$, $K_{SD}$, and $K_{SP}$ are defined in Refs. 1 and 3.

| | $V_{1S_0}$ | $V_{2S_0}$ | $\bar{V}_{1P_{012}}$ | $V_{1D_2}$ | $L$ | $K_{SD}$ | $K_{SP}$ |
|---|---|---|---|---|---|---|---|
| Pure H.O. | 718 | 598 | 479 | 383 | 586 | 36 | 36 |
| Correlated | 794 | 506 | 485 | 390 | 611 | 19 | 33 |
| Corr. + Pauli | | | | | | | |
| $E_D$ (MeV) = 10 | 793 | 603 | 485 | 389 | 627 | 36 | 43 |
| 30 | 761 | 588 | 482 | 387 | 608 | 34 | 39 |
| 50 | 744 | 581 | 481 | 386 | 597 | 33 | 36 |
| 70 | 734 | 576 | 480 | 386 | 592 | 32 | 35 |

suppression (in the hard-core region). Moreover, we have repeated the calculations with the Coulomb interaction, $(e^2/r)f(r)$, which is due to two protons with charge distribution of finite size[8] instead of $e^2/r$. The values obtained for the relative $S$, $P$, and $D$ states are smaller than those given in Table I by ~35, 5, and 1 keV, respectively. We thus conclude that including the effect of the finite size of the proton charge distribution we obtained values which are even smaller than those which correspond to point proton interaction $e^2/r$ and uncorrelated wave functions. Finally, we mention here that similar calculations were carried out for the nuclear $2s$-$1d$ ($A \sim 18$), $2p$-$1f$ ($A \sim 42$) shells[9] leading to the same conclusions that the calculated Coulomb matrix elements are too small to account for the experimental values. This is in accordance with the Nolen-Schiffer problem,[8] which exists throughout the Periodic Table, that in the present available theory the calculated Coulomb displacement energies are smaller than the experimental results.

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