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Classical view of the application of sum rules to inelastic form factors*

G. F. Bertsch
Department of Physics and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824
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Classical arguments are given to show why the Tassie model of form factors can be derived from a doorway hypothesis.

Deal and Fallieros[1] have made an interesting application of sum rules to transition densities, following the work of Noble[2] and Kao and Fallieros.[3] These sum rules enable one to relate transition densities $\rho_{n0}(x)$ between ground and excited states to the ground state density distribution ρ_0 and the transition moments of simple operators M_{n0}. Expressed in coordinate space, the sum rules are of the form [1]

$$\sum_n E_n \rho_{n0}(x) M_{n0} = \tfrac{1}{2}\langle[\rho,[H,M]]\rangle = \frac{\hbar^2}{2m}\vec{\nabla}\cdot(\vec{\nabla}M)\rho_0|_x . \quad (1)$$

Here E_n is the excitation energy of the nth state, H is the Hamiltonian, and M is an isoscalar operator, such as $r^L Y^L(\hat{r})$. The interesting use of the sum rule is based on the hypothesis that one state only has $M_{n0} \neq 0$; Fallieros calls this state the "doorway." Then Eq. (1) can be solved for ρ_{D0} as

$$\rho_{D0}(x) = \frac{\hbar^2}{2m}\frac{\vec{\nabla}\cdot(\vec{\nabla}M)\rho_0}{M_{D0}E_D} . \quad (2)$$

The usual operators are $r^L Y^L$ for L-pole transitions and r^2 for monopole transitions. Then the predicted ρ_{D0} is the same as in the Tassie model,[4] which works quite well in describing inelastic electron form factors.[5]

The above relation, Eq. (2), has seemed somewhat mysterious to the author; it appears as though a great deal of physics is being extracted from very little input. The following classical derivation may make the relation more intuitively plausible, and also show how the doorway hypothesis replaced dynamical assumptions. While classical systems are usually described by second-order differential equations in time, it is often convenient to use two first-order equations, the equation of continuity, and a dynamical equation of motion. As we will see, only the equation of continuity plays a role in the derivation. We consider a system in its ground state, described by a density ρ. An impulsive field acts on the system at time $t=0$. Let us call the field

$$V(r,t) = M(r)\delta(t) . \quad (3)$$

There will be a force on the particles given by

$$\vec{F} = -\vec{\nabla}V = m\vec{a} ,$$

which may be integrated to obtain the velocity of the particles just after $t=0$ as

$$\vec{v} = -\frac{\vec{\nabla}M}{m} . \quad (4)$$

Further integration to finite times would require knowledge of the internal forces on the system, which have nonzero resultants when displacements become finite. The further integration is obviated by the assumption that the system rings harmonically at frequency $\omega = E_D/\hbar$. The equation of continuity for $t>0$ may then be expressed as

$$-\vec{\nabla}\cdot\rho\vec{v} = \vec{\nabla}\cdot(\vec{\nabla}M)\rho/m = \frac{d\rho}{dt} = \omega\delta\rho , \quad (5)$$

where $\delta\rho(x)$ is the amplitude of the density oscillation at x.

To make the final connection with the quantum formula, we have to express $\delta\rho$ and M as expectation values with state vectors. The vectors are

$$|\psi\rangle = |\psi_0\rangle \quad t<0 ,$$

$$|\psi\rangle = |\psi_0\rangle + C_{D0}e^{i\omega t}|\psi_D\rangle \quad t>0 .$$

Then

$$\delta\rho \sin\omega t = (C_{D0}e^{i\omega t} + C^*_{D0}e^{-i\omega t})\rho_{D0} . \quad (6)$$

We find C_{D0} from

$$\hbar i \frac{d}{dt}|\psi\rangle = V|\psi\rangle ;$$

$$C_{D0} = \frac{M_{n0}}{i\hbar} . \quad (7)$$

Equation (2) is then a simple consequence of Eqs. (5), (6), and (7).

In a few words, the Tassie model emerges because it is based on irrotational and incompressible velocity fields, and operators $r^L Y^L$ generate just this type of field.

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