TABLE II. Comparison with calculations.

Isotopes	T_z	$M-A$ (MeV) Experimental	$M-A$ (MeV) Garvey *et al.* revised[a]	$M-A$ (MeV) Garvey *et al.* 1969[b]	E_n (MeV) Experimental	E_n (MeV) Garvey *et al.* revised[a]	E_n (MeV) Garvey *et al.* 1969[b]
^{27}Na	$\frac{5}{2}$	-5.88 ± 0.14	-5.98	-6.65	7.10 ± 0.14	7.10	7.10
^{28}Na	3	-1.26 ± 0.17	-1.55	-3.38	3.45 ± 0.22	3.64	4.80
^{29}Na	$\frac{7}{2}$	2.73 ± 0.21	0.86	-2.14	4.08 ± 0.27	5.66	6.84
^{30}Na	4	8.37 ± 0.46	6.89	2.72	2.43 ± 0.51	2.04	3.21

[a]See Ref. 11. [b]See Ref. 10.

lines will be obtained in the future, using the technique that is described here for the first time.

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Nuclear Response Function

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A fast numerical method is described for calculating nuclear excitation properties with δ-type interactions. The method is applied to ^{208}Pb, and the following properties of the nucleus are found: (i) The quadrupole strength has two main pieces, one a low state and one identifiable as the giant quadrupole; (ii) excitations with $L>2$ do not seem to have high-energy collective parts; (iii) the giant dipole $L=1, T=1$ is too low unless the interaction has a strong momentum dependence.

There has been much interest recently[1-3] in Hartree-Fock calculations of nuclei using the parametrization of the nucleon interaction in terms of momentum and density-dependent δ functions, called Skyrme interactions.[4] If the Hartree-Fock description of a ground state is reasonable, the random-

phase approximation (RPA) should give a good description of the collective excitations. We describe here a fast method for calculating the RPA response of nuclei to excitation operators. Conventionally, a Hamiltonian matrix is diagonalized to find the eigenstates of the system, and then the matrix elements of the operator are evaluated between the ground and the excited states. Our technique is to evaluate the RPA Green's function directly in coordinate space by matrix inversion. The size of the matrix is determined by the mesh in coordinate space, rather than the number of configurations included in the excited state. This gives a considerable savings in the dimensionality of the matrices. The use of coordinate space directly in particle-hole calculations was first done by Blomqvist.[5]

Let us take a two-body interaction of the form

$$V(\vec{r}_1,\vec{r}_2)=\sum_L V_{\alpha\beta}{}^L(r_1,r_2)Y^L(\hat{r}_1)Y^L(\hat{r}_2)M_\alpha(1)M_\beta(2), \tag{1}$$

where the operator set M can include $\vec{\sigma}$, τ, and ∇, as well as the unit operator. We also define a set of two-particle Green's functions with the multipole expansion

$$G_{\alpha\beta}{}^{(0)}(\vec{r}_1,\vec{r}_2,\omega)=\sum_L G_{\alpha\beta L}{}^{(0)}(r_1,r_2,\omega)Y^L(\hat{r}_1)Y^L(\hat{r}_2), \tag{2}$$

$$G_{\alpha\beta L}{}^{(0)}(r_1,r_2,\omega)\equiv\sum_{ph} C_\alpha{}^{phL}(r_1)C_\beta{}^{phL}(r_2)\left[\frac{1}{\epsilon_p-\epsilon_h-i\eta-\omega}+\frac{\pi_\alpha\pi_\beta}{\epsilon_p-\epsilon_h+\omega-i\eta}\right]. \tag{3}$$

The sum is over unoccupied orbits p (particle) and occupied orbits h (hole), with the Hartree-Fock orbitals and energies denoted by φ^i and ϵ_i. The function C is defined as

$$C_\alpha{}^{phL}(r)=\int d\Omega\, Y^L(\hat{r})M_\alpha[\varphi^{j_p}(\vec{r})\varphi^{j_h}(\vec{r})]^L \tag{4}$$

and π_α is -1 or $+1$ depending on whether or not $C_\alpha{}^{phL}$ reverses sign under interchange of the roles of particle and hole. For the case $M=1$, we have simply

$$C_1{}^{phL}\equiv\left(\frac{(2j_p+1)(2j_h+1)}{4\pi(2L+1)}\right)^{1/2}(j_p\tfrac{1}{2}j_h-\tfrac{1}{2}|L0)\varphi_{\text{radial}}{}^{j_p}(r)\varphi_{\text{radial}}{}^{j_h}(r). \tag{5}$$

For $M=\vec{\sigma}$, the coefficient C is given in Ref. 5; the rather lengthy expression for $M=\nabla$ will be given in a future publication.

In RPA theory, the response to an operator

$$\mathfrak{F}_\alpha{}^L=\sum f(r)Y^L(r)M_\alpha \tag{6}$$

is given by[6,7]

$$\sum_{\text{states } k}\langle 0|\underline{\mathfrak{F}}^L|k\rangle^2\delta(E_k-E_0)=\pi^{-1}\,\mathrm{Im}\langle\underline{\mathfrak{F}}^L\underline{G}^{\mathrm{RPA}}\underline{\mathfrak{F}}^L\rangle=\pi^{-1}\,\mathrm{Im}\langle\underline{\mathfrak{F}}^L\underline{G}_L{}^{(0)}(\underline{1}-\underline{V}^L\underline{G}_L{}^{(0)})^{-1}\underline{\mathfrak{F}}^L\rangle. \tag{7}$$

We have suppressed the indices α,β using a matrix notation for $\mathfrak{F}$, G, and V. This expression does not include exchange, but for δ interactions, exchange can be included exactly in V, once the spin- and isospin-flip parts of the Green's function have been introduced.[5] The integrals over coordinates implied in Eq. (7) are replaced by sums over coordinate mesh points, reducing the problem to one of matrix inversion.

The appropriate two-body interaction for RPA calculations is given by the second derivative of the Hartree-Fock potential-energy functional,[8,9]

$$V_{12}=\sum_{t,s}\tfrac{1}{2}[1+(-1)^{t-t'}\vec{\tau}_1\cdot\vec{\tau}_2]\tfrac{1}{2}[1+(-1)^{s-s'}\vec{\sigma}_1\cdot\vec{\sigma}_2]\frac{\delta\mathcal{U}_{[\rho]}}{\delta\rho_{ts}\,\delta\rho_{t's'}}, \tag{8}$$

where $\mathcal{U}_{[\rho]}$ is the potential-energy functional in the Hartree-Fock theory, and ρ_{ts} is the operator for the density of nucleons with $\tau_z=t$ and $\sigma_z=s$. Using the parametrization of the interaction given by Brink and Vautherin, having constants t_0, t_1, t_2, and t_3, one obtains

$$\begin{aligned}V_{12}=\delta(r_{12})\{&\tfrac{3}{4}t_0+\tfrac{3}{8}t_3\rho+\tfrac{3}{16}t_1[-(\nabla_1{}^2+\nabla_1{}'^2+\nabla_2{}^2+\nabla_2{}'^2)-(\nabla_1-\nabla_1')\cdot(\nabla_2-\nabla_2')+(\nabla_1+\nabla_1')\cdot(\nabla_2+\nabla_2')]\}\\&\times[1-\tfrac{1}{3}\vec{\tau}_1\cdot\vec{\tau}_2-\tfrac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2-\tfrac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\tau_1\cdot\tau_2]-\delta(r_{12})\{\tfrac{1}{4}t_2[\nabla_1{}^2+\nabla_1{}'^2+\nabla_2{}^2+\nabla_2{}'^2+(\nabla_1-\nabla_1')\cdot(\nabla_2-\nabla_2')]\\&+\tfrac{3}{4}t_2(\nabla_1+\nabla_1')\cdot(\nabla_2+\nabla_2')\}[\tfrac{5}{4}+\tfrac{1}{4}\vec{\tau}_1\cdot\vec{\tau}_2+\tfrac{1}{4}\vec{\sigma}_1\cdot\vec{\sigma}_2+\tfrac{1}{4}\vec{\sigma}_1\circ\vec{\sigma}_2\vec{\tau}_1\circ\vec{\tau}_2].\end{aligned} \tag{9}$$

TABLE I. Energy and octupole strength of the lowest 3^- state in ^{208}Pb in various approximations. The interaction is renormalized by a factor Z to make the spurious 1^- state degenerate with the ground state.

Interaction	Mesh size (fm)	M_α set	Z	Energy (MeV)	$\sum r^3Y^3$
Skyrme I	1	$1,\nabla^2$	0.803	2.4	0.58
Skyrme I	0.5	$1,\nabla^2$	0.810	2.4	0.60
Skyrme I	1	$1,\nabla^2,\vec{\sigma},\tau$	0.820	2.2	0.67
Skyrme II	1	$1,\nabla^2$	1.002	2.3	0.80
Skyrme II	1	$1,\nabla^2,\nabla$	1.008	2.8	0.49
Experimental				2.614	0.4

The interaction between two particle-hole states is given by matrix elements of an operator in coordinate 1 which annihilates one of the particle-hole states and an operator in coordinate 2 which creates the second particle-hole state. The gradient operators ∇_i and ∇_i' act on the particle and the hole coordinates, respectively. In the matrix element for the density-dependent term, t_3, we have assumed that the neutron and proton densities are the same.

The dimensionality of the Green's function matrix becomes excessive if all operators M_α are included. The operator ∇^2 can be calculated with the Green's function for $M=1$ using

$$\nabla_p^2+\nabla_h^2=(2m/\hbar^2)[U_p(r)+U_h(r)-\epsilon_p-\epsilon_h].$$

Aside from this, we usually calculated the response with only the diagonal matrix elements of G in this operator space. Calculations with extended operator sets do not affect the gross structure of the response, but can influence the position and strength of the lowest collective states considerably. This is illustrated in Table I. We now describe our calculational procedure and the results for ^{208}Pb.

First we obtain the eigenstates of a particle in a Woods-Saxon well, placed in a spherical box of radius 14 fm. Thus the continuum is replaced by a discrete set of orbits. We find all occupied orbits and all unoccupied having up to two more radial nodes or three more angular nodes than the corresponding occupied orbits. We expect the wave functions to approximate the Hartree-Fock orbitals adequately. However, the energies must be modified to match the Hartree-Fock energies given by Brink and Vautherin.[5] This is done crudely using the expectation of $\langle\nabla^2\rangle$ and $\langle\rho\rangle$ for each orbital.

Calculations were done with interactions Skyrme I and II of Ref. 2. Both interactions give correct nuclear size and binding but have different character otherwise. Skyrme II is very momentum dependent, and gives an effective mass $m^*\sim 0.6$, while Skyrme I achieves saturation by a strong density dependence, and has a single-particle spectrum much like the Woods-Saxon well. In principle, RPA theory gives a spurious 1^- state degenerate with the ground state, but in fact, because of our approximations we had to adjust the strength of the interaction to achieve this. This required a negligible change in Skyrme II and a -20% change in Skyrme I for the numerical approximations discussed below. We used these adjusted interactions in the calculation of the response.

The Green's function must be evaluated at each energy of interest. We replace ω by $\omega+i\Gamma/2$ with $\Gamma\approx 1$ MeV to smooth the distribution and evaluate G in steps of 0.5 MeV. This gives us the gross structure, and we can refine the mesh in the neighborhood of particular states of interest, such as the lowest 3^- collective state. For most calculations, we took a mesh of 1 fm spacing and cut off the particle-hole configuration space at an energy of 30–40 MeV. Typically there are 100–300 configurations and a point takes 15 sec to run on our Σ-7 computer.

Some general conclusions which have emerged so far are the following:

(a) The low-lying collective states are fairly independent of the approximation and the choice of interaction. This is illustrated in Table I, where we quote the energy of the 3^- state and its mass multipole strength $\sum r^3Y^3$. While the energies seem reasonable, the transition strengths are too large by 75%.

(b) Collective states at higher energy depend strongly on the interaction. This was also shown for lighter nuclei by Zamick.[8] Recently, a possible observation of a giant quadrupole state at

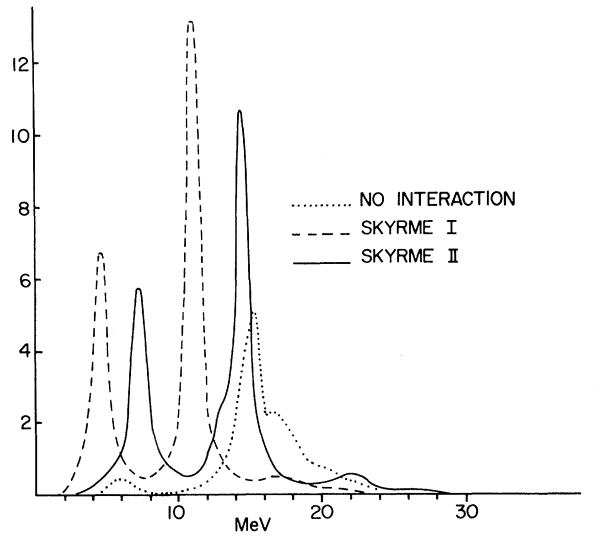

FIG. 1. Quadrupole strength in ^{208}Pb in units of $10^3/\pi$ fm^4/MeV. Dotted curve, with the free Green's function $G^{(0)}$; dashed and solid curves, for the Skyrme interactions with the diagonal G^{RPA}. Mesh spacing, 1 fm.

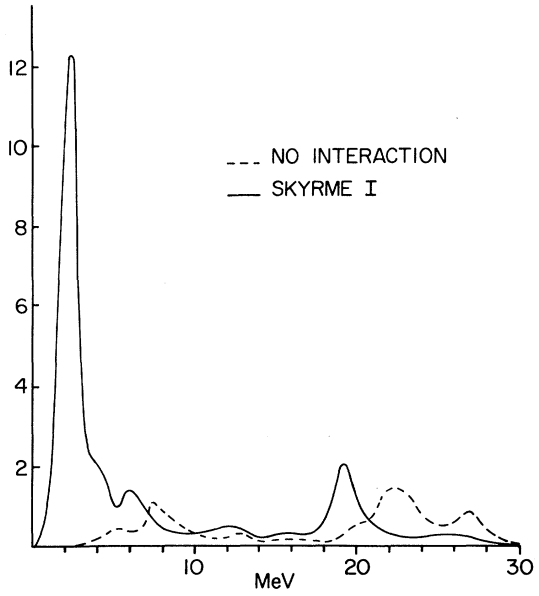

FIG. 2. Octupole strength in ^{208}Pb for Skyrme I interaction in units of $10^5/\pi$ fm^6/MeV. Dashed curve, with $G^{(0)}$; solid curve, with the diagonal G^{RPA}. Mesh spacing, 1 fm.

an energy of 10–12 MeV, has generated much interest.[10,11] As seen in Fig. 1, Skyrme I yields such a state, but Skyrme II places it at much higher excitation. For $L>2$, there seems to be no high-lying collective states, although remains of shell structure still persist, as is seen in the $L=3$ strength plotted in Fig. 2.

(c) According to conventional wisdom, the breathing mode (with $T=0, L=0$) is a collective state pushed up in energy by the nuclear incompressibility. We find no evidence of this; the strength for the breathing operator[12]

$$3\rho + r\,d\rho/dr$$

is at about the same energy for both G^{RPA} and $G^{(0)}$, ~18 MeV with Skyrme I.

(d) The giant dipole state ($T=1, L=1$) lies at 13.5 MeV experimentally,[13] but "realistic" forces tend to predict it a couple of MeV too low.[14] We find that Skyrme I fails as miserably as the realistic forces, but Skyrme II gives an energy of 14.5 MeV, somewhat too high.

More detailed derivations and results will be given in a future publication.

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